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Summary:

Fresh surface water is becoming increasingly scarce worldwide. This increased groundwater use as the most cost-effective source of water. Groundwater over-extraction, however, could result in another environmental externality in the form of land subsidence (LS). In the long run, LS may alter the geological formation of the aquifer. The loss of aquifer storage capacity owing to LS is one such negative externality that is seldom discussed. By investigating the indirect loss of the aquifer storage capacity due to LS along with the other direct LS negative externalities, we developed a dynamic economic optimization model for groundwater utilization and evaluated various policy interventions (quota and taxes). We found that taxes on land sinking and aquifer storage capacity reduction have a significant effect on withdrawals and water table levels in sediment-based aquifers compared to fractured-rock aquifers. Quotas are more beneficial to farmers than taxes in terms of social welfare benefits. When it comes to groundwater conservation, taxes are better than quotas.

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Abstract

Fresh surface water is becoming increasingly scarce worldwide. This increased groundwater use as the most cost-effective source of water. Groundwater over-extraction, however, could result in another environmental externality in the form of land subsidence (LS). In the long run, LS may alter the geological formation of the aquifer. The loss of aquifer storage capacity owing to LS is one such negative externality that is seldom discussed. By investigating the indirect loss of the aquifer storage capacity due to LS along with the other direct LS negative externalities, we developed a dynamic economic optimization model for groundwater utilization and evaluated various policy interventions (quota and taxes). We found that taxes on land sinking and aquifer storage capacity reduction have a significant effect on withdrawals and water table levels in sediment-based aquifers compared to fractured-rock aquifers. Quotas are more beneficial to farmers than taxes in terms of social welfare benefits. When it comes to groundwater conservation, taxes are better than quotas.

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Keywords: land subsidence; groundwater over-extraction; aquifer storage capacity; taxes; quotas.

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1. Introduction

Land subsidence (LS) is defined as "... sinking of the earth's surface arising from the subsurface movement of earth materials" (Galloway et al., 1999, p.1). Land subsidence is a global problem with economic and environmental negative effects (Herrera-Garcia et al., 2021; Kok and Costa, 2021). One major cause of LS, amongst several others, is the long-term over-extraction of groundwater (Saber et al.,2018). Previous evidence shows that land subsidence caused by groundwater withdrawal stems from the compaction of the soil particles (Galloway et al., 1998; Wade et al., 2018; Minderhoud et al., 2017). As a result, the ground particles shrink and compact to fill up the formed pores, and the ground surface sinks.

As the ground surface sinks, it poses greater damage to infrastructures, the performance of groundwater-dependent ecosystems, and the aquifer storage capacity. For instance, the formation of earth fissures, damages to buildings, failure in well casings, and damages to municipal infrastructures like drains, roads, pipelines, bridges, railways, sewers, and municipal dams are examples of such effects (Faunt et al., 2015, Galloway et al., 2016; Conway, 2015; Moteahd et al., 2019). Particularly, infrastructures such as buildings, roads, bridges, and dams start to crack which later leads to their walls and surfaces breaking apart or deforming. According to Galloway et al., (1999, p.1), more than four-fifths of the overall land subsidence in the United States has been attributed to excessive groundwater mining. Furthermore, a recent survey by the United States Geological Survey (USGS) California Water Science Center (Sneed, 2018) revealed that land subsidence stemming from groundwater over-exploitation can not be reversed in most cases. Governments spend substantial financial resources on fixing the aforementioned negative effects. Therefore, to halt land subsidence, optimal groundwater extraction paths coupled with land subsidence averting strategies should be incorporated into local water management strategies.

Three types of externalities are associated with groundwater over-extraction: (1) Congestion/water table depth externality (depth); (2) Direct Land Subsidence (LS) damage externality (infrastructure, ecosystems); and (3) Indirect loss of the aquifer storage capacity due to LS (Dinar et al., 2020). The depth externality entails the groundwater depletion effects on pumping costs for all groundwater consumers. Over-extraction of groundwater raises the depth (or height) to the water table, causing water to be pumped to the surface from deeper levels. This necessitates the use of additional energy, making future extractions more expensive. The direct land subsidence externality entails impacts, such as infrastructure and ecosystem damages. As previously stated, infrastructures suffer damage (i.e., cracking and deforming) as the soil surface on which they are sitting sinks. Furthermore, groundwater over-extraction results in water scarcity and quality deterioration, which has an impact on aquatic ecosystems that rely on aquifers for survival. Groundwater over-extraction and land subsidence also impair or affect the ecological processes and state of groundwater-dependent ecosystems. The indirect loss of storage due to LS entails the opportunity cost of losing the aquifer's storage capacity over time. There is a state of compaction in which the pore spaces filled by the sinking soil are lost forever. Even if the aquifer is recharged, it will not be capable of holding the same amount of water as before. Many previous studies have examined the depth externality (Gisser and Sanchez, 1980; Brill and Burness, 1994; Guilfoos et al., 2013; de Frutos Cachorro et al., 2014; Tomini, 2014; Allen and Gisser, 1984). Several other papers looked at ecosystem damages as an externality from groundwater extraction, not considering land subsidence (Esteban and Albiac, 2011; Roumasset and Wada, 2013; Esteban and Dinar, 2016). Dinar et al. (2021) provided a list of additional environmental damages that have been pointed at in previous literature. Previous research, not necessarily economics, focused only on three aspects of LS, namely LS prediction (Wade, 2016; Mahmoudpour et al., 2016; Shen and Xu, 2011; Wang et al., 2019), LS monitoring (Saber et al., 2018; Minderhoud et al., 2017; Galloway et al., 1998; Ljungdahl, 2015; Cao et al., 2020; Wright et al., 2004; Khorrami et al., 2019), and evaluating and quantifying the direct LS economic damages (Wade et al., 2018; Costa et al., 2020; Hu et al., 2013; Yoo and Perrings, 2017).

The literature about the economic analyses of LS and its management is scant. Only a few studies have economically assessed LS impacts (Warren et al., 1975; Lixin et al., 2010; Yoo and Perrings, 2017; Wade et al., 2018; Borchers and Carpenter, 2014). It is therefore important to initiate further research activities that shed more light on such economic implications to address the vast gap that currently exists in the literature. This paper contributes to such a body of literature. We build on Dinar et al (2020) that examined the direct LS externality together with the indirect loss of storage externality due to LS. Our contribution lies in our clear inclusion of the groundwater drawdown patterns, instead of previous studies that rather considered groundwater extraction patterns (Dinar et al., 2020). This is important, given the difference that often occurs between groundwater extraction and drawdown patterns. We use a subsidence function to determine the subsidence rate, using groundwater drawdown. Furthermore, we assume that the water extractor (i.e: farmer) is penalized for actions leading to the aquifer's storage capacity reduction, which is her water withdrawals. This was done by charging the extractor with a fixed (Pigouvian) tax per every m of inelastic compaction. We follow Leake and Galloway, (2007, p.2) and define compaction as a reduction in the thickness of sediments due to an increase in vertical compressive stress. The vertical compressive stress within the sediments is increased as groundwater is extracted from aquifers, resulting in compaction. When inelastic compaction occurs, pore space is permanently lost and cannot be restored. This means that the aquifer's storage capacity is lost forever. Therefore, the larger the inelastic compaction, the larger the loss to aquifer storage capacity.

In order to investigate the direct LS negative externality, we design a direct-damage function that is dependent on the rate of subsidence caused by groundwater drawdown. This allows groundwater extractors to be held accountable for all direct LS negative repercussions caused by groundwater abstraction from the aquifer. For the indirect LS negative externality, we create an indirect-damage function that is dependent on the degree of inelastic compaction caused by groundwater extraction from the aquifer. Likewise, this allows groundwater extractors to be held accountable for all groundwater withdrawals that lead to the reduction in the aquifer's storage capacity. Therefore, we evaluate the various trajectory changes in the water table elevation and farmers' groundwater withdrawals as a result of the imposed damage functions, and hence incorporate policy interventions to regulate such externalities. Taxes and quotas on pumping are investigated as policy interventions. The effects of pumping taxes and quotas on social welfare are discussed and compared using the net present value framework.

This paper is organized as follows: Section 2 describes the dynamic optimization model we developed for the LS-considered groundwater management. We present several propositions that provide a general solution of the model (Their proof appears in the Appendix). In Section 3, we apply the model empirically to the Dendron aquifer in South Africa and describe the data-collection process. Then in Section 4, we describe the policy interventions used to regulate the pumping from the aquifer. In Section 5, we provide and discuss the results of the empirical application. Finally, in Section 5, we conclude and discuss policy implications.

2. The model

Our point of departure is a farmer who owns a water pump in a certain agricultural area (AA) overlaying a certain groundwater aquifer. We assume, for simplicity and without loss of generality, that groundwater is the only source of water available for irrigation.[4] Assuming that there is a regulator in charge of the aquifer, this farmer is subject to charges put forward to avert all the negative externalities stemming from groundwater depletion. Such penalties (e.g., monetary charges per meter of compaction) reduce extractions as the farmer wouldn't want to be charged more than her marginal benefit from the water extracted. Following Gisser and Sanchez (1980), the demand for irrigation water pumped by this farmer is given by Equation (1) below

$$W(t) = g + kP, \;\; g > 0, k < 0, \tag{1}$$

where $W(t)$ is the groundwater extraction rate at each point in time t, g and k are parameters of the water demand function, and P represents the price of irrigation water. The inverse demand function for Equation (1) is given by Equation (2) below (removing the operator t for convenience)

$$P = \frac{W}{k} - \frac{g}{k}. \tag{2}$$

The pump owner's total revenue is given by Equation (3) below

[4] Urban sector extractions lead to similar LS results but are not modelled here to keep the setup simple.

$$\int_W P(W)dW = \frac{W^2}{2k} - \frac{gW}{k}. \tag{3}$$

Following Gisser and Sanchez (1980) and Esteban and Albiac (2011), we denote the extraction costs by the following cost function given in Equation (4) below

$$\tilde{P} = C_0' + C_1'(S_l - H) \tag{4}$$

where $(S_l - H)$ is the height to the water table, with S_l and H representing the irrigated field surface elevation and the water table elevation levels, respectively. The parameters C_0' and C_1' represent the fixed costs and the marginal extraction cost, respectively. Reformulating Equation (4), we obtain a revised cost function equation $\bar{P} = C_0 + C_1H$, where C_0 and C_1 are the intercept and the water table coefficient respectively, defined as $C_1 = -C_1'$ and $C_0 = C_0' + C_1'S_l$. Therefore, the pump owner's overall benefit is given by the difference between total revenue and total extraction costs as defined by the expression below

$$\frac{W^2}{2k} - \frac{gW}{k} - (C_0 + C_1H)W. \tag{5}$$

Taking into consideration *the direct LS externality*, we define the rate (in meters) at which the land is sinking due to groundwater pumping as suggested by Wade et al (2018)

$$Sink(D(W)) = \eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W) \tag{6}$$

where W, η, b, ψ, ε, and $D(W)$ represent the total groundwater extractions in cubic meters (m^3), the density of water in Kg/m^3, the aquifer's thickness in meters (m), the confining material compressibility in ms^2/kg, the acceleration due to gravity in m/s^2, and the drawdown due to pumping in m, respectively. As a result, the social cost arising from the direct LS negative externality is given by $\beta Sink(W)$, where β represents the monetary cost of this negative externality ($\$/m$ of sinking). Dinar et al., (2020) substituted W for $D(W)$ in Equation (6), which could be problematic, as groundwater extraction might be different from groundwater drawdown. This paper aims to address this problem and also introduce a more accurate method of measuring the loss of the storage capacity externality due to LS. Pumping externalities associated with groundwater resources, according to various studies, are spatially and temporally variable (Wade et al., 2018; Merrill and Guilfoos, 2017). For simplicity and without loss of generality, we assume a single-cell aquifer with a non-heterogeneous distribution of impacts and wells, and that the direct LS externality is evenly distributed throughout the aquifer and all pumping wells. The only action of the pumper that affects the change in the water table is withdrawal (W), as the natural recharge and percolation are not determined by the pumper but by some natural phenomenons, $D(W)$ is given by the drawdown formula (Thiem Equation) in Equation (7) below

$$D(W) = \frac{W}{2\pi T} \ln(\frac{r_i}{r_w}) \tag{7}$$

where T, r_w, and r_i represent the aquifer's transmissivity in $m^2/year$, the radial distance from the pumping well[5] in m, and the radius of influence[6] in m, respectively. The foremost common way of finding the radius of influence is by using empirical formulas. Hence, r_i is given by Lembke's (1887) formula in Equation (8) below

$$r_i = b\sqrt{\frac{v}{2Z}} \tag{8}$$

where v and Z represent the aquifer's hydraulic conductivity in $m/year$, and the groundwater recharge rate ($m^3/year$), respectively. The latter refers to the rate at which surface water is added to an aquifer (Sharma, 1986). We assume that surface water is composed of the aquifer's natural recharge and other runoff, like percolation and injection wells (or water injected as part of a managed aquifer recharge program). The other negative externality associated with LS, the indirect loss of aquifer's storage capacity is addressed as follows. We define the rate (in meters) at which inelastic compaction is occurring due to groundwater pumping (Construction outlined in the Appendix) as suggested by Poland (1969, p.288-290)

$$\Delta q = y_v b \gamma_w (1 - n + n_w) D(W), \tag{9}$$

where y_v represents the compacting beds' mean compressibility. The parameters γ_w, n, and n_w represent the unit weight of water (N/m^3), the porosity (dimensionless), and the moisture content of sediments above the water table (in the unsaturated zone) as a fraction of total volume (dimensionless). As explained in the Appendix, any rise in the effective stress (of course, lower than the pre-consolidation stress) causes elastic compaction. Hence, in practice, the precise point in time at which elastic compaction begins is not known with certainty until a certain level of subsidence occurs. We hypothesize that the elastic compaction phase begins exactly at

[5] Radial distance from well (Radius of investigation) refers to the maximal distance from a pumping well up to which the properties of an aquifer have a significant impact on drawdown at the well (Bresciani et al., 2020).

[6] Radius of influence (Influence radius) refers to the maximal distance from a pumping well up to which the impact of pumping is significant. (Bresciani et al., 2020).

the same time ($t = 0$) with the planning horizon. This assumption becomes even more important when the model is applied to aquifers that have already experienced or are experiencing land subsidence at present. Therefore, during the elastic compaction phase, previous Equation (5) becomes

$$\frac{W^2}{2k} - \frac{gW}{k} - (C_0 + C_1 H)W - \beta(\eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W)). \quad (10)$$

We further assume, following Dinar et al., (2020), that inelastic compaction begins at a certain point (a threshold H_T) in the water table drawdown. This is the point at which the effective stress exceeds the pre-consolidation stress. Thus, no aquifer storage capacity is lost before reaching this point. Likewise, we denote by ϕ the economic cost of losing the aquifer's storage capacity ($\$/m$ of inelastic compaction). The larger the inelastic compaction, the larger the aquifer's storage capacity loss. As a result, during inelastic compaction, the pump owner's overall benefit (Equation (5)) is given by Equation (11). In the model, we impose a fixed tax (ϕ) on the water extractor as a penalty for contributing to LS's third externality, as illustrated in the last term of Equation (11).

$$\frac{W^2}{2k} - \frac{gW}{k} - (C_0 + C_1 H)W - \beta(\eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W)) - \phi \Delta q. \quad (11)$$

The equation of motion of the aquifer water table level $\dot{H} = \frac{1}{AS}[R - (1-\alpha)W],\ 0 < t < +\infty$ has been widely used in economic works pertaining to groundwater management (see Koundouri et al., 2017). Where A, S, R, and $0 \leq \alpha < 1$ represent the area of the aquifer (m^2), the storativity coefficient (dimensionless), the natural recharge rate ($m^3/year$), and the percolation return flow coefficient (dimensionless), respectively. We also hypothesize that managed aquifer recharge (MAR) is allowed in our model for the sake of simplicity and applicability. Therefore, after combining (10), (11) and the water table dynamics, the social welfare maximization problem of the pump owner becomes

$$\max_{W,H,t_T} \int_{t=0}^{t=t_T} e^{-it}[\frac{W^2}{2k} - \frac{gW}{k} - (C_0 + C_1 H)W - \beta(\eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W))]dt$$

$$+ \int_{t=t_T}^{\infty} e^{-it}[\frac{W^2}{2k} - \frac{gW}{k} - (C_0 + C_1 H)W - \beta(\eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W)) - \phi \Delta q]dt, \quad (12)$$

subject to

$$\dot{H} = \begin{cases} \frac{1}{AS}[R-(1-\alpha)W] & t < t_T \\ \frac{1}{AS}[R-(1-\alpha)W] & t \geq t_T \end{cases} \tag{13}$$

$$H(t_0) = H_0,\ H(t_T) = H_T,\ t_T = \text{ free.} \tag{14}$$

In (12) - (14), i represents the discount rate. The value for H_T and t_T are determined by the model. A two-stage optimal control resolution is used to solve the above optimization problem. The theoretical foundations of the two-stage optimal control resolution were studied by Tomiyama(1985) and Tomiyama and Rossana (1989), and recently several authors have employed this approach (Makris, 2001; Boucekkine et al., 2004; de Frutos Cachorro et al., 2014; Esteban and Dinar, 2016; Dinar et al., 2020). The method requires splitting the optimization problem into two sub-problems. The second sub-problem ($SP2$) follows below.

$$\max_{W_2,H_2,t_T} \int_{t=t_T}^{\infty} e^{-it}[\frac{W_2^2}{2k} - \frac{gW_2}{k} - (C_0 + C_1H_2)W_2 - \beta(\eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W_2)) - \phi\Delta q]dt \tag{15}$$

subject to

$$\dot{H}_2 = \frac{1}{AS}[R-(1-\alpha)W_2], \tag{16}$$

$$H_2(t_T) = H_T;\ \ H_T, t_T \text{ given.} \tag{17}$$

Land subsidence occurs as a result of compaction, as previously stated. There are two types of compaction: elastic and inelastic. Elastic compaction is reversible, but inelastic compaction is not. The elastic compaction phase is said to always occur first and then it switches to inelastic compaction at a time t_T. Proposition 1 presents the optimal solution during the inelastic compaction phase, assuming that the elastic compaction phase switches to the inelastic compaction phase at a time t_T.

Proposition 1. (Optimal solution) *if the farmer maximizes (15) subject to (16 - 17), the optimal paths ($W_2^*(t)$ and $H_2^*(t)$) are determined by the following expressions.*

$$W_2^*(t) = \frac{x_2AS}{\alpha-1}[H_0 - \frac{NN-i\frac{R}{\alpha-1}}{ikC_1}]e^{x_2(t-t_T)} - \frac{R}{\alpha-1}, \tag{18}$$

$$H_2^*(t) = [H_0 - \frac{NN - i\frac{R}{\alpha-1}}{ikC_1}]\mathrm{e}^{x_2(t-t_T)} + \frac{NN - i\frac{R}{\alpha-1}}{ikC_1}, \tag{19}$$

where

$$x_2 = \frac{i - \sqrt{i^2 - 4 \cdot \frac{ikC_1(\alpha-1)}{AS}}}{2}$$

$$NN = -ig - ikC_0 - ik\beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T} \ln(\frac{r_i}{r_w})$$
$$-ik\phi \cdot by_v\gamma_w(1 - n + n_w)\frac{1}{2\pi T} \ln(\frac{r_i}{r_w}) + \frac{C_1 kR}{AS}.$$

A proof of this proposition can be found in the Appendix.

When only the water table depth externality is considered, Gisser and Sanchez (1980) obtained the optimal water table level and optimal extractions. Both the optimal water table level and optimal extractions have additional terms that depend on the subsidence rate and the quantity of inelastic compaction, compared to Gisser and Sanchez (1980) optimal solutions. Two new terms, $-ik\beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T} \ln(\frac{r_i}{r_w})$ and $-ik\phi \cdot by_v\gamma_w(1 - n + n_w)\frac{1}{2\pi T} \ln(\frac{r_i}{r_w})$, are introduced in both expressions. The economic cost of land sinking (direct LS negative externality) and the economic cost of losing aquifer storage capacity influence both the optimal levels of the water table and extractions. The greater the economic cost of each of the two LS negative externalities, the greater the influence on the water table and extractions at their optimal levels. It's worth noting that the terms $-ik\beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T} \ln(\frac{r_i}{r_w})$ and $-ik\phi \cdot by_v\gamma_w(1 - n + n_w)\frac{1}{2\pi T} \ln(\frac{r_i}{r_w})$, respectively, represent the meters at which the land sinks per amount of groundwater extracted and the meters of inelastic compaction experienced per amount of groundwater extracted. Hence, for effective and optimal groundwater management policies, a better understanding of the contribution of groundwater pumping to land sinking and inelastic compaction of the aquifer is required. As a result, we arrive at propositions 2 and 3.

Proposition 2. *The economic cost of land sinking has a direct impact on how groundwater* *extractions are managed over time. If $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the higher the economic cost of land sinking the lower the optimal level of extractions. If $i > \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the higher the economic cost of land sinking the higher the optimal level of extractions. Furthermore, the economic cost of land sinking has a direct impact on how the water table level is managed over time. If $e^{x_2(t-t_T)} > 1$, the higher the economic cost of land sinking the lower the optimal level of the water table. If $e^{x_2(t-t_T)} < 1$, the higher the economic cost of land sinking the higher the optimal level of the water table.*

A proof of this proposition can be found in the Appendix.

Proposition 3. *The economic cost of losing an aquifer's storage capacity has a direct impact on how groundwater extractions are managed over time. If $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the higher the economic cost of losing the aquifer's storage capacity the lower the optimal level of extractions. If $i > \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the higher the economic cost of losing the aquifer's storage capacity the higher the optimal level of extractions. Furthermore, the economic cost of losing aquifer's storage capacity has a direct impact on how the water table level is managed over time. If $e^{x_2(t-t_T)} > 1$, the higher the economic cost of losing the aquifer's storage capacity the lower the optimal level of the water table. If $e^{x_2(t-t_T)} < 1$, the higher the economic cost of losing aquifer's storage capacity the higher the optimal level of the water table.*

A proof of this proposition can be found in the Appendix.

Propositions 2 and 3 demonstrate that a deeper understanding of particular aquifer properties is required to determine the two economic costs (cost of land sinking and cost of losing aquifer storage capacity) , which result in decreased extractions and higher water table levels over time. The social discount rate, the slopes of the given water demand and pumping costs, the aquifer's area and storativity coefficient, and the aquifer's return flow coefficient are all factors to consider. These findings have tangible policy implications for groundwater management, particularly in aquifers where LS is not taken into account in groundwater management policies or where the LS-associated costs are determined by using a different approach. Our results demonstrate, in a way that has never been done before, how the rate of subsidence and the decline in aquifer storage capacity influence optimal groundwater management.

Once the solution to sub-problem 2 is obtained, we solve for a solution to sub-problem 1 (SP_1). Following Raouf et al., (2003); Boucekkine et al., (2004); and Dinar et al., (2020), we impose the following matching conditions for optimality and continuity.

$$\lambda_1(t_T) = -\frac{\partial SP_2^*(t_T, H_T)}{\partial H_T} \tag{20}$$

$$\mathcal{H}_1^*(t_T, H_T) = \frac{\partial SP_2^*(t_T, H_T)}{\partial t_T}, \tag{21}$$

where $SP_2^*(\cdot)$ represents the optimal solution to sub-problem 2. The variable $\mathcal{H}_1$ represents the hamiltonian for sub-problem 1. As a result, sub-problem 1 is given by

$$\max_{W_1, H_1} \int_{t=0}^{t=t_T} e^{-it} \left[\frac{W_1^2}{2k} - \frac{gW_1}{k} - (C_0 + C_1 H_1)W_1 - \beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \cdot D(W_1)\right] dt + SP_2^*(H_T^*, t_T^*) \tag{22}$$

subject to

$$\dot{H}_1 = \frac{1}{AS}[R - (1-\alpha)W_1], \tag{23}$$

$$H_1(t_0) = H_0;\ \ H_1(t_T) = H_T;\ H_0 \ \text{ given and } \ H_T \ \text{ free}; 0 \le t < t_T. \tag{24}$$

In Proposition 1, we found the optimal solution for the inelastic compaction phase. This phase occurs only if the elastic compaction phase switches at time t_T; otherwise, the elastic compaction phase continues indefinitely. Proposition 4 presents the optimal solution during the elastic compaction phase, assuming that the elastic compaction phase ends at time t_T and the inelastic compaction phase begins after that.

Proposition 4. (Optimal solution) *if the farmer maximizes (22) subject to (23 - 24), the optimal paths ($W_1^*(t)$ and $H_1^*(t)$) are determined by the following expressions.*

$$W_1^*(t) = \tilde{A}\mathrm{e}^{y_1 t} + \tilde{B}\mathrm{e}^{y_2 t} - \frac{R}{\alpha-1}, \tag{25}$$

$$H_1^*(t) = \frac{(\alpha-1)}{AS\, y_1}\tilde{A}\mathrm{e}^{y_1 t} + \frac{(\alpha-1)}{AS\, y_2}\tilde{B}\mathrm{e}^{y_2 t} + \frac{N-\frac{iR}{\alpha-1}}{ikC_1}, \tag{26}$$

where

$$y_{1,2} = \frac{i \pm \sqrt{i^2 - 4\cdot\frac{ikC_1(\alpha-1)}{AS}}}{2}$$

$$N = -ig - ikC_0 - ik\beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T}\ln\left(\frac{r_i}{r_w}\right) + \frac{kC_1 R}{AS}.$$

$$\tilde{A} = \frac{y_1 AS}{\alpha-1}\left[H_0 - \frac{N-\frac{iR}{\alpha-1}}{ikC_1} - \frac{\left[H_0-\frac{N-\frac{iR}{\alpha-1}}{ikC_1}\right]\mathrm{e}^{y_1 t_T} - \left[H_T-\frac{N-\frac{iR}{\alpha-1}}{ikC_1}\right]}{\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T}}\right],$$

$$\tilde{B} = \frac{y_2 AS}{\alpha-1}\left[\frac{\left[H_0-\frac{N-\frac{iR}{\alpha-1}}{ikC_1}\right]\mathrm{e}^{y_1 t_T} - \left[H_T-\frac{N-\frac{iR}{\alpha-1}}{ikC_1}\right]}{\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T}}\right].$$

A proof of this proposition can be found in the Appendix.

The economic cost of land sinking affects both the optimal level of the water table and levels of extractions, as found under the inelastic compaction phase. The greater the economic cost of land sinking, the greater the influence on the water table and extractions at their optimal levels. Similarly, the social discount rate, the water demand slope, the pumping costs slope, the aquifer's area and storativity coefficient, and the aquifer's return flow coefficient are the aquifer properties to be considered when determining the economic cost of land sinking that yields lower extractions and higher levels of the water table. This results is presented in Proposition 5.

Proposition 5. *The economic cost of land sinking has a direct impact on how groundwater extractions are managed over time. The higher the economic cost of land sinking the lower the* *optimal level of extractions, except when* $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$ *and* $e^{y_1 t_T} < e^{y_2 t_T}$. *Furthermore, the economic cost of land sinking has a direct impact on how water table level is managed over time. If* $e^{y_1 t_T} > e^{y_2 t_T}$, *the higher the economic cost of land sinking the lower the optimal level of the water table. If* $e^{y_1 t_T} < e^{y_2 t_T}$, *the higher the economic cost of land sinking the higher the optimal level of the water table.*

A proof of this proposition can be found in the Appendix.

In this study, two regulatory policies (quotas and taxes) on groundwater utilization are investigated. All of the previous propositions only addressed taxes as a policy intervention to address land subsidence impacts. For quotas, we introduce an additional constraint ($W(t) \leq \widehat{W}(t)$) to the constraints (13) and (14). Where $\widehat{W}(t)$ represents the quota level. The social welfare maximization problem ((12) - (14)) should then be resolved with this new additional constraint in mind. We don't need to introduce a Lagrangian function to determine the necessary conditions since the constraints on the control variable ($W(t)$) are independent of the state variable ($H(t)$) and do not change with time t. Proposition 6 presents the condition under which $W^*(t) \leq \widehat{W}(t)$ is satisfied.

Proposition 6. *if the farmer maximizes (12) subject to (13) - (14) and an additional constraint* $W(t) \leq \widehat{W}(t)$, *the optimal paths (*$W^*(t)$ *and* $H^*(t)$*) should satisfy the following expression.*

$$t_T \geq \frac{1}{x_2} \ln\left[\frac{x_2 AS[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]}{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)}\right] \tag{27}$$

A proof of this proposition can be found in the Appendix.

Proposition 6 above represents the case when both taxes and quotas are taken into account. When quotas are implemented, we expect that the height to the water table level will fall as more water is left untouched due to withdrawal limits. As a result, the rate of compaction is lowered. Consequently, the transition (time t_T) from the elastic to the inelastic phase of compaction is delayed. All of the optimal paths ($W^*(t)$ and $H^*(t)$) described in Propositions 1 and 4 contain the switching time t_T (either implicitly or explicitly). Therefore, the switching time given in equation (27) is substituted into equations (18) and (19). For equations (25) and (26) where t_T appears implicitly, we compute the value of H_T using t_T to determine the optimal paths. Likewise, Proposition 7 presents the case when only quotas are taken into account.

Proposition 7. *if the farmer maximizes (12) where $\beta = \phi = 0$ subject to (13) - (14) and an additional constraint $W(t) \leq \widehat{W}(t)$, the optimal paths ($W^*(t)$ and $H^*(t)$) should satisfy the following expression.*

$$N_0 \geq H_0 - \frac{[\widehat{W}(\alpha-1)+R]ikC_1}{x_2 AS} + \frac{iR}{\alpha-1} \tag{28}$$

A proof of this proposition can be found in the Appendix.

The theoretical results in propositions 1-7 are demonstrated using an empirical application to the Dendron aquifer in South Africa. We follow with a sensitivity analysis of the parameters associated with land subsidence externality as well as the groundwater storage capacity externality. This enables us to have a clear interpretation of the optimal paths as they are more complex to interpret in the theoretical framework.

3. Application to the Dendron aquifer, South Africa

The numerical application uses the Dendron aquifer in the Hout River Catchment of South Africa, located in the Limpopo River Basin. The land usage in the Dendron (known as Mogwadi) area is predominantly for agriculture, as evidenced by the presence of more than 200 farms in the area, 50 of which are entirely irrigated and do not support livestock (Masiyandima et al., 2002). The remaining farms are livestock-based, and as a result, groundwater is used for livestock and feed watering. Because there are no reliable surface water sources in the area, groundwater has become the area's sole source of water. For more than 2 decades, the aquifer has been the sole source of irrigation water for both commercial and non-commercial farmers in the area

(Masiyandima et al., 2002). The area had around 335 boreholes in 1986, with irrigation accounting for 95% of groundwater withdrawals (Jolly, 1986). The remaining 5% of groundwater withdrawals were for domestic consumption and livestock watering. Groundwater overexploitation in the aquifer is said to have begun in the 1970s, when the area had a lower density of farms and hence fewer irrigation activities than it has now. The problem worsened as additional farms were established in the area, and also exacerbated by the recurring drought events in recent years. The irrigated area in the Dendron area rose by nearly 170 percent between 1968 and 1986, leading to a 133 percent rise in groundwater withdrawals and a 5 to 35-meter drop in the water table (Masiyandima et al., 2002). According to Masiyandima et al. (2002), the farmers' union set a regulation that only 3% of each 1000 hectares of land should be irrigated with groundwater, in an attempt to prevent overexploitation of the aquifer. In addition, farmers began practicing a variety of cropping patterns and irrigation water management strategies, such as switching from furrow irrigation to manual move sprinkler systems, and finally center pivots, which are now utilized on the majority of farms in the area (Masiyandima et al., 2002). As a result, around early 1990s, groundwater levels began to rise again in the aquifer (Masiyandima et al., 2002). Severe flood events, in combination with the aforementioned farming patterns and irrigation water management practices, induced a rise in the water table level. Severe flood events have been observed in the Limpopo River Basin in the last ten years, in 1955, 1967, 1972, 1975, 1977, 1981, 1990, 2000, and 2013 (CRIDF, 2018).

Dendron farmers are also required to apply for a permit for each borehole used for irrigation under the country's 1998 National Water Act, with the exception of household use, which is not licensed (NWA 1998). The Department of Water and Sanitation (DWS) monitors groundwater abstractions as per each individual permit during its validity. Groundwater permits are valid for forty years and are reassessed every fifth year (NWA 1998). However, the height to water table continued to increase further due to the failure to monitor groundwater abstractions, weak or non-enforcement of groundwater permits, and farmers' failure to comply with mitigation techniques put in place to prevent groundwater overexploitation (Fallon et al., 2018). In 2000, the height to water table increased at a much faster rate after regulatory measures were implemented, resulting in a 50 to 100-meters below ground level (Masiyandima et al., 2002). In June 2012, a borehole drilled into the aquifer revealed a height to water table of 98 meters below ground level (Fallon et al., 2018). Apart from the obvious occurrence of land subsidence events in the Dentron area, land subsidence events have been recorded in the Limpopo province of South Africa, where the Dentron area is located (Oosthuizen and Richardson, 2011, p.8).

3.1 Data for the numerical application

The hydrological and economic data used in our empirical application are gathered from previous studies in the area as well as groundwater reports from the South Africa Department of Water and Sanitation (DWS). The Hout River Catchment has a semi-arid climate, with an average annual rainfall of 407 millimeters, sandy soil, with Luvisols covering approximately 56% of the catchment (Ebrahim et al., 2019). Geologically, the Dendron aquifer is made up of two interdependent aquifers, the upper weathered aquifer and the lower fractured aquifer (Jolly, 1986). According to Murray and Tredoux (2002), the aquifers are partially filled with clay as a result of weathering in the upper aquifer. The presence of fine-grained sediments (clay sediments) within the aquifers makes the Dendron aquifer more vulnerable to land subsidence episodes. The upper aquifer area is reported to be between 12-50 meters below the ground surface, while the lower aquifer area extends up to 120 meters below the ground surface. The contact between the upper and lower aquifer, according to Masiyandima et al. (2002), is at 35-50 meters below ground surface. As a result, the total aquifer thickness is assumed to be 108 meters. The catchment's irrigated field surface elevation is roughly 1289.5 meters above sea level (Ebrahim et al., 2019). We further assume that the initial height to the water table is 98 meters below surface level, which is the aquifer's most recent water table measurement (Fallon et al., 2018). The long-term annual natural recharge to the aquifer is estimated to be 3.8 percent of MAP, or 7.35 million cubic meters, according to Dziembowski (1976) and Jolly (1986). The aquifer's annual groundwater recharge (overall recharge of groundwater with surface water via river infiltration, rain infiltration, underground supply, returns from surface water uses etc.) is estimated at 702 million cubic meters (Holland, 2011). The aquifer covers an area of about 1, 600km^2 (Masiyandima et al., 2002). The aquifer's return flow coefficient is calculated to be 0.2 (Jolly, 1986). Ebrahim et al. (2019) reported the mean hydraulic conductivity to be 259.15 meters per year (0.71 meters per day). When the hydraulic conductivity is multiplied by the aquifer thickness, the transmissivity is 27, 988.2 square meters per year. Fractured rocks have a vertical compressibility of $3.3 \times 10^{-10} ms^2/kg$ to $6.9 \times 10^{-10} ms^2/kg$ (Domenico and Schwartz, 1990). We assume that the confining material compressibility is equal to $5.1 \times 10^{-10} ms^2/kg$ by using the median of the aforementioned compressibility values. Likewise, the mean compressibility of compacting beds is considered to be the same. The radius of influence is equal to 46.4 meters. The radial distance from the well (r_w) is generally given by $r_w = \frac{1}{2} r_i$ as a rule of thumb, that is $r_w = 23.2m$ (Bresciani et al., 2020).

In comparison to the lower aquifer, which has a high yield, the upper aquifer has a low storage and yield (Jolly, 1986). Furthermore, the water table level is reported to be deeper than in the weathered zone in most parts (Holland, 2012). As a result, the majority of agricultural production wells in the aquifer are drilled up to the aquifer's lower zone, which is the lower fractured aquifer (Fallon et al., 2018; Holland 2011, 2012). Therefore, the yield-related parameters for the aquifer are determined using the lower aquifer's hydrological data. According to Masiyandima et al. (2002), the storativity coefficient is 0.0025. When certain yield-related parameters are unavailable, other authors have used values from other weathered-fractured aquifers to analyze groundwater in the Dendron area in the past (Jolly 1986; Ebrahim et al., 2019). We made use of a porosity value of 0.0068 calculated by Stober and Bucher (2007) for a fractured zone of a weathered-fractured aquifer in Germany.

The economic price and cost values are given in US dollars for simplicity. The slope of the irrigation water demand function (decrease in quantity demanded per one dollar rises in irrigation water price) in the adjacent sub-catchment, the Middle Olifants sub-catchment, is equal to -0.181 (Walter et al., 2011). The price of irrigation water is 3907.38 US dollars (27, 000 Rands) per million cubic meters, based on the 2011 currency rate (Lange and Hassan, 2006). This figure represents the average tariff for raw water in the catchment area. We calculate the intercept of the demand function to be 724.24 using the average groundwater abstractions from the aquifer of 17 million cubic meters per year (Ebrahim et al., 2019). According to DWAF (2003), the fixed pumping cost in a fractured rock aquifer of depths up to 120 meters below ground surface in South Africa is 4, 551.20 US dollars per cubic meter. This figure represents the fixed cost of operating and maintaining a single pump (or borehole), that is, the cost when no groundwater is pumped. This covers mechanical and electrical maintenance, as well as the amortization of extraction technology. The electrical expenses to pump water from the aquifer are estimated to be 0.0026 USD per cubic meter or 2,604.92 US dollars per million cubic meters (DWAF, 2003). To maintain the same reference year for the parameter values as 2011, the pumping cost intercept in 2011 is 5,209.84 US dollars. Using the pumping cost function, we find that the slope of the pumping cost function in the area is -13.29. During the reference year (2011), the social discount rate was set at 8% (Conningarth Economists, 2014, pp.69-70). At the time, this was the official social discount rate applicable in South Africa. In the analysis, we use the same discount rate to calculate the net present value (NPV) of the calculated welfare. However, the level of the social discount rate is important in deciding how profitable policy interventions are for farmers. Because 8% is relatively high, we utilize 2.3 percent to perform a sensitivity analysis and analyze the influence of the discount rate level on the NPV. This rate (2.3 %) represents the average social discount rate in South Africa across the years as simulated by Addicott et al (2020).

Table 1: Hydrological and economic values of the Dendron aquifer.

Parameter	Description	Units	Value (US dollars)
k	Water demand slope	$\$/Mm^3$	-0.181
g	Water demand intercept	$\$/Mm^3$	724.24
C_0	Pumping costs intercept	$\$/Mm^3$	5209.84
C_1	Pumping costs slope	$\$/Mm^3\ m$	-13.29
α	Return flow coefficient	dimensionless	0.2
H_0	Current water table	m	98
R	Natural recharge	$Mm^3/year$	7.35

A	Aquifer area	km^2	1600
S	Storativity coefficient	dimensionless	0.0025
i	Social discount rate	%	0.08
β	Economic cost of land sinking	$\$/Mm^3$	2943.95
η	Water density	Kg/m^3	1000
b	Aquifer's thickness	m	108
ψ	Confined material compressibility	ms^2/kg	5.1×10^{-10}
n	Porosity	dimensionless	0.0068
Z	Groundwater recharge rate	$Mm^3/year$	702
T	Transmissivity	$m^2/year$	27988.2
v	Hydraulic conductivity	$m/year$	259.15
r_w	Radial distance from well	m	23.2
r_i	Influence radius	m	46.4
ϕ	Economic cost of losing storage capacity	$\$/Mm^3$	31.57
y_v	Compacting beds mean compressibility	ms^2/kg	5.1×10^{-10}
ε	Gravitational acceleration	m/s^2	9.81
n_w	Vadose moisture/ Total volume	dimensionless	0.1
γ_w	Unit weight of water	N/m^3	9810

4. Policy interventions

Two policy interventions taxes and quotas on pumping are used in our empirical application to regulate the negative externalities associated with LS. For the taxes, we hypothesize that pumping should be taxed in order to minimize groundwater extractions and therefore the negative impacts of LS. The specific tariff per meter of elastic compaction was obtained using the existing fixed charge for infrastructure costs per abstraction (volume of groundwater extracted) for a typical river basin in South Africa. For the inelastic compaction phase, we assumed that the tax is the same as the current afforestation cost per volume of groundwater extraction charge. The charges for infrastructure costs and afforestation per abstraction costs for a typical river basin in South Africa were 2,943.95 USD per million cubic meters and 31.57 US dollars per million cubic meters, respectively, during the reference year (Conningarth Economists, 2014, p.105). As a result, without loss of generality, we assume that the economic costs of the direct LS negative externality and the indirect LS negative externality are 2,943.95 US dollars per meter of sinking and 31.57 US dollars per meter of inelastic compaction, respectively. We used multiple values derived by altering the assumed fixed taxes for the sensitivity analysis to assess the impact of both the economic cost of land sinking and the cost of losing aquifer storage capacity on the water table level and extractions. We used $\beta = 2{,}943.95$ and $\beta = 10{,}000$ for the economic cost of land sinking. This allowed us to see the effects of extremely high or low tax values of the direct LS negative externality on the water table level and withdrawals. We used $\phi = 31.57$ and $\phi = 63.14$ for the economic cost of losing aquifer storage capacity (Conningarth Economists, 2014, p.105). This also allowed us to observe the effects of high or low tax values of the indirect LS negative externality.

We use the existing allocated pumping quotas to regulate pumping as per the DWS. In South Africa, groundwater extractors are required to apply for a permit for each borehole or pump used for the extraction of water for irrigation under the country's 1998 National Water Act, with the exception of household use, which is not licensed (NWA 1998). The Department of Water and Sanitation (DWS) monitors groundwater abstractions as per each permit during its validity. Groundwater extraction for uses other than domestic consumption requires either a general authorization (GA) or a water use licence from DWS (Reddick and Kruger, 2019). To be eligible for a groundwater permit, the user must meet all of the requirements outlined in the applicable permit. For the GA, a farm can only extract 0.04 million cubic meters per year, regardless of its location (Reddick and Kruger, 2019). As a result, we assume that each farmer's maximum pumping allotment (quota) is 0.04 million cubic meters per year. We use 0.02 million cubic meters per year for the sensitivity analysis on quotas.

5. Results and discussions

The analysis uses 2011 as the base year because it is the most recent year for which we have data on the water table level. Since the steady-state levels are reached between the 30th and 50th years of our simulation, the planning horizon is 50 years. First, we consider a situation without land subsidence and any regulatory policy (Figure 1). We observe a sharp increase in groundwater extractions and the height to water table for the first 10 years, then a gradual increase thereafter. The increase in groundwater extractions is due to the rising population growth and economic activities in the area. The increase in the height of the water table is due to the water table depth externality; the higher the volume of groundwater extraction, the deeper the water table, and hence the greater the extraction cost. After 33 years, groundwater extractions attain a steady-state ($9Mm^3$), while the water table level reaches a steady-state ($117\ m.b.g.l$) after 34 years. Although this time frame appears to be far too long to reach a steady-state in an overexploited aquifer like the one under analysis, multiple earlier studies in the literature reveal that a steady-state might be reached after 5000 years or more (Gisser and Sanchez, 1980; Allen and Gisser, 1984). However, Jolly (1986) indicated that little or no groundwater is found after 120 meters below ground level in the Dendron aquifer. This water table level is almost reached in the first 34 years of pumping, indicating that the aquifer is being over-pumped, as shown in Figure 1. Social welfare is defined as the net present value (NPV) when all of the LS negative externalities are taken into account. The farmer's private welfare is represented by the private NPV (when only the depth externality is considered). The farmer's private NPV is positive (330,302.5 US dollars), suggesting that the revenue value exceeds the costs. This profit is made, however, at the expense of draining the aquifer in 34 years. This finding indicates that the aquifer is being over-exploited, necessitating prompt intervention. Also note that groundwater extractions are negative during the first three years, indicating that aquifer replenishment is occurring, maybe through artificial recharge or significant rainfall events that lasted for short periods (flash floods). During this time, however, the height of the water table continued to increase, indicating that a drought was occurring over the long term.

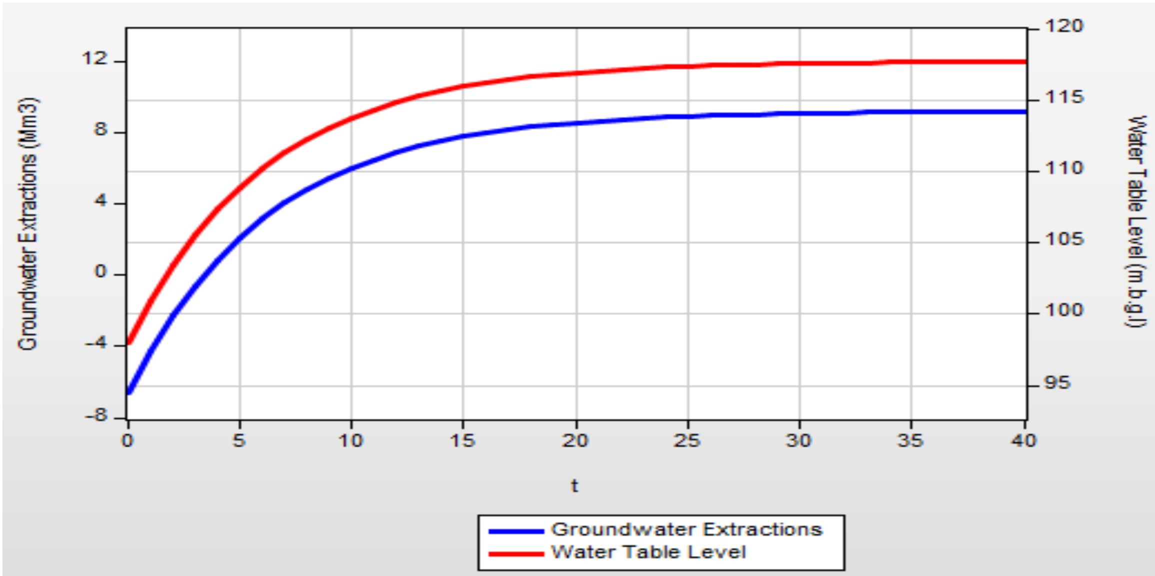

Figure 1. Dendron aquifer without LS and without any policy intervention.

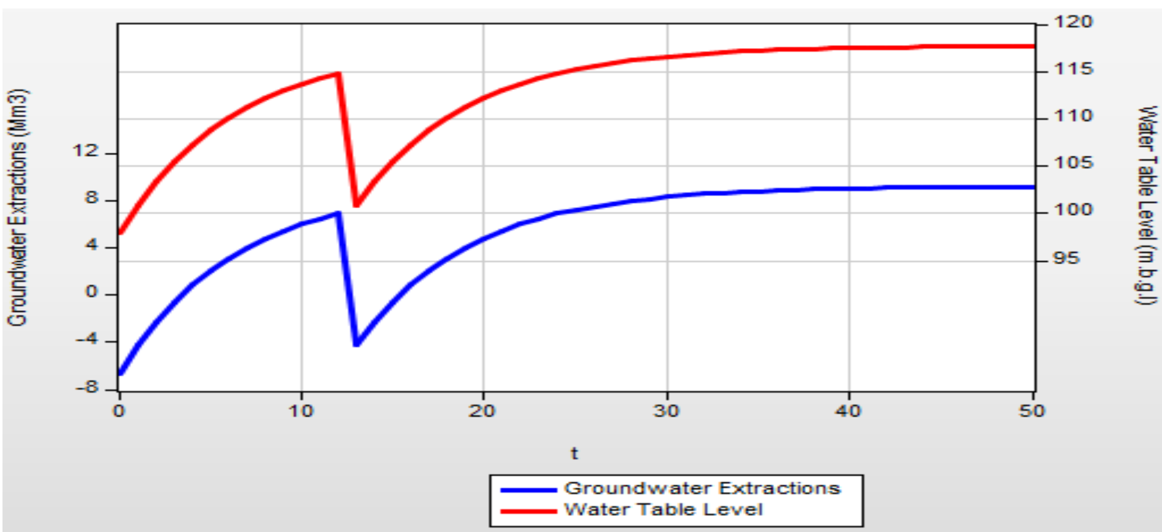

Figure 2. Dendron aquifer with taxes on land subsidence.

Secondly, we consider a situation in which there is land subsidence and taxes to avert land subsidence impacts (Figure 2). The only policy intervention considered is groundwater pumping levies, which are implemented to reduce land subsidence impacts. Thus, we use all of the parameters associated with the indirect loss of aquifer storage capacity externality and the direct land subsidence externality listed in Table 1 ($\beta, \eta, \psi, n, Z, T, v, r_w, r_i, \phi, y_v, \varepsilon, \tilde{n}_w, \gamma$). We see that the height to the water table drops from 114 meters below ground level to 101 meters below a ground level around year 12. As a result, during the same period, groundwater extractions decreased from 7 Mm3 to -4 Mm3. The negative value (-4 Mm3) indicates that aquifer replenishment is occurring around year 12. Particularly, in year 13, the critical threshold (H_T) is reached, and the height to water table level abruptly rises again. As farmers begin to adapt to the new land sinking taxes, the height to the water table begins to rise. Farmers lower extractions initially when taxes are implemented in year 12, but after they have adapted to the new taxes, they gradually increase extractions to fulfill their water needs. This is because, although farmers are charged with taxes on groundwater extractions, population growth and economic activity continue to rise. It's worth mentioning that, from the beginning (year 0) until year 12, both the water table level and groundwater extractions are roughly the same as they are when the direct LS externality is not considered. This demonstrates that penalizing groundwater extractors for the direct LS externality does not affect groundwater extractions or water table elevation. This result, however, only applies to aquifers with rock-based lithologies, such as the Dendron aquifer. Because aquifers with rock-based lithologies are less prone to LS (Dinar et al., 2021), the same result may differ when sediment-based lithologies, which are more prone to compaction, are analyzed. The aquifer's susceptibility to LS is determined by the compacting beds' mean compressibility (or the confined material compressibility) y_v. To demonstrate this, we will use a hypothetical aquifer with a higher compressibility value. We adjust y_v from 5.1×10^{-10} to 0.05 to represent the hypothetical aquifer with sediment-based lithology or compacting beds, which are more compressible than rock-based lithologies. In this case, when groundwater extractors are charged or penalized for land sinking, groundwater extractions decrease as shown in Figure 3. Charging groundwater extractors for the direct LS externality could be beneficial if the current water table level was within the upper aquifer of the Dendron aquifer, which has sediment-based lithologies. In the year 43, the water table reaches its steady-state level. This result demonstrates that charging groundwater extractors for the indirect loss of aquifer storage capacity increases the water table level while also increasing groundwater extractions. The social NPV is negative (-801,052.61 US dollars), suggesting that the revenue value is below the costs. As a result, when land subsidence taxes are in place, farmers lose money. However, the loss is made at the expense of supporting the aquifer to attain the water table level steady state (117 m.b.g.l) in year 43 rather than year 34 (when LS is not considered). This minimizes aquifer depletion over a short period, as well as the effects of land subsidence. Over a 50-year planning horizon, the social net present value for the hypothetical aquifer that is extremely prone to land subsidence adds up to 1,696,061,155,669,340 US dollars. This suggests that land subsidence taxes reduce the ostensible effects while also enhancing long-term profits for farmers.

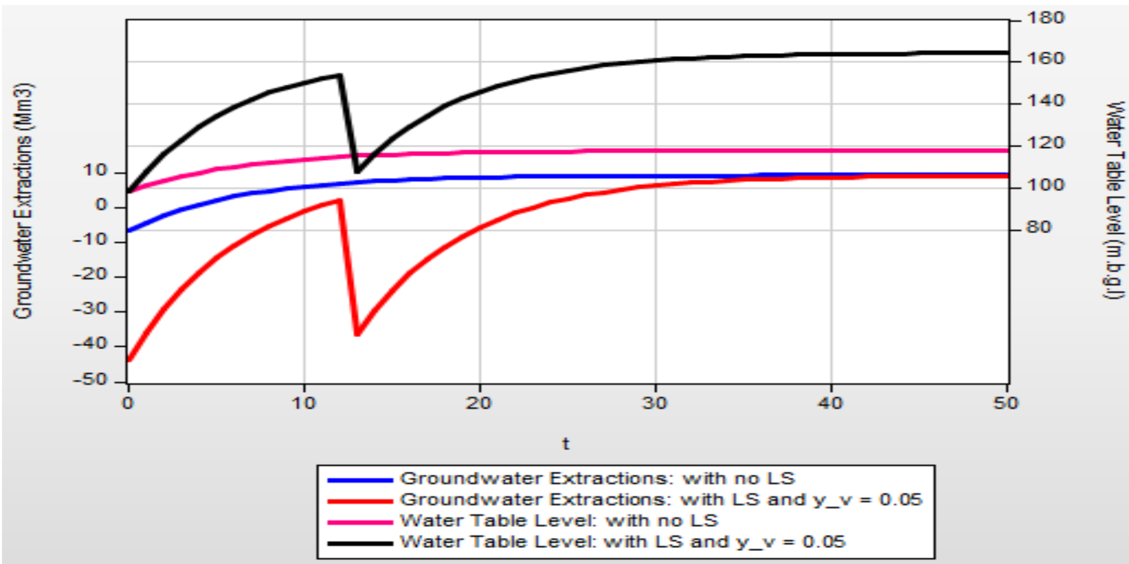

Figure 3: Dendron aquifer with taxes on LS and a high compressibility value of the compacting beds.

To estimate the impacts of LS on the optimal paths for groundwater extraction and water-table level, we perform a sensitivity analysis on the economic costs imposed on groundwater extractors to avert land sinking and the reduction of aquifer storage capacity. Except for the transmissivity (T) and the radius of influence (r_i), all of the parameters associated with the two damage functions are multiplicative. There is no need to perform a sensitivity analysis on the radial distance from the pumping well (r_w) because it is computed from r_i. As a result, except for T and r_i, the result of the sensitivity analysis performed with the economic cost of each damage function applies to all other parameters of that damage function.

Figure 4 confirms what we already know; charging groundwater extractors for the direct LS externality has no effect on the water table level in the Dendron aquifer. The same can be said for groundwater extractions. As previously stated, this is due to the compacting beds in the aquifer having a lower compressibility value. Even if very higher values of β are taken (i.e. 150,000), the water table level does not change.

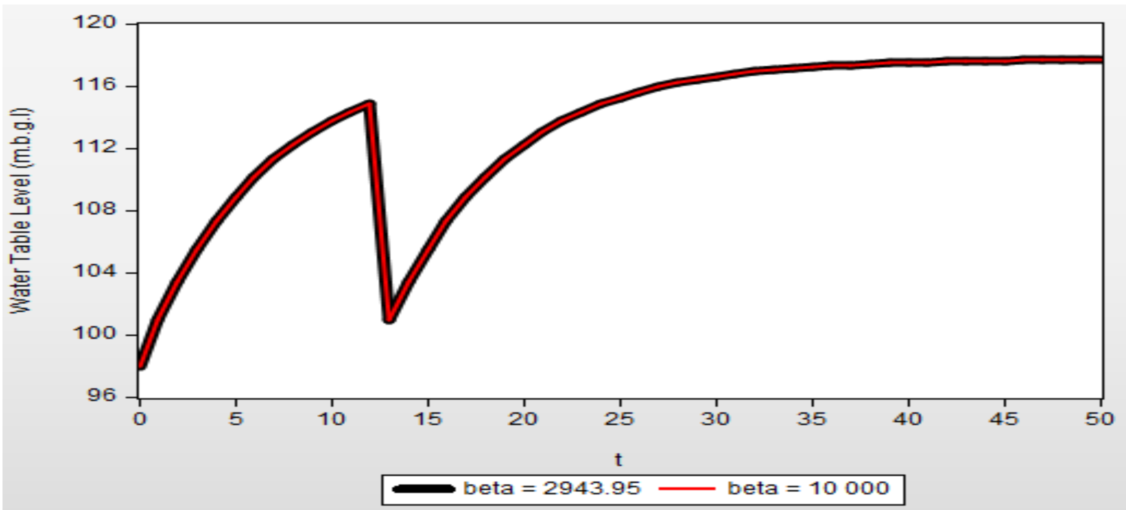

Figure 4: Dendron aquifer with taxes on LS and different values of the economic cost of land sinking.

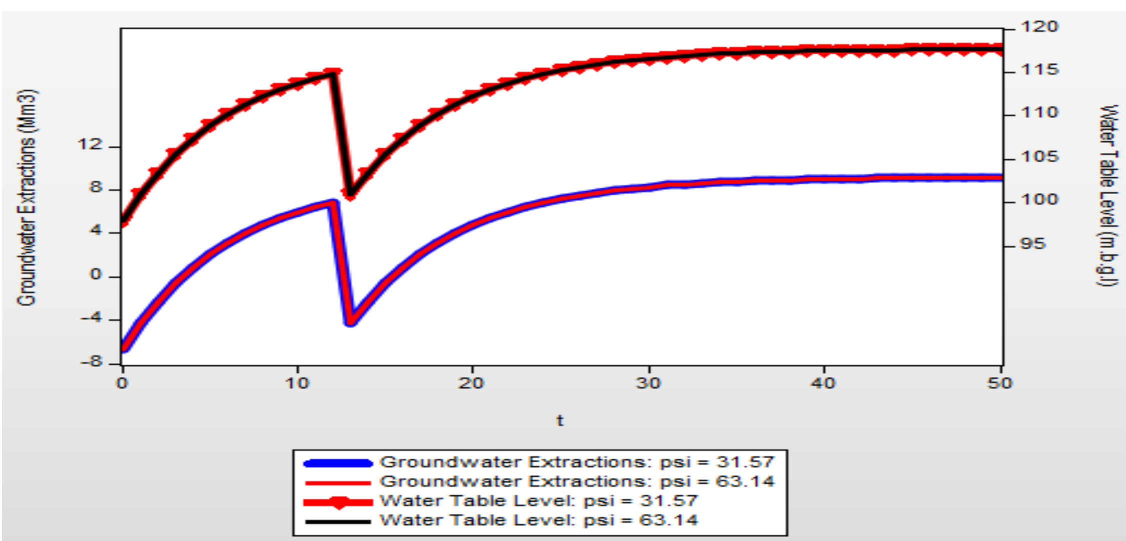

Figure 5: Dendron aquifer with taxes on LS and different values of the economic cost of losing aquifer storage capacity.

When groundwater extractors are taxed for the indirect LS negative externality, the same effect of rock-based lithologies affects groundwater extractions and the water table level (Figure 5). Only that, compared to the zero effect reported for the direct LS negative externality, leads to a very small effect in this case. When groundwater extractors lead to the indirect loss of aquifer storage capacity, as shown in Figure 5, the height to the water table drops sharply. Because the water table is near the irrigation surface, groundwater users react rapidly by extracting more water than previously. However, regardless of whether they are charged more or less for the same negative externality, their withdrawal patterns (which are dependent on the water table level) stay the same. To summarize, holding groundwater extractors liable for the Dendron aquifer's indirect loss of storage capacity delays the aquifer's depletion, but nothing can be done once the first delay occurs. By the first delay, we mean the farmers' initial reaction to the imposed tax, which differs from how they will react to the same tax in the long run.

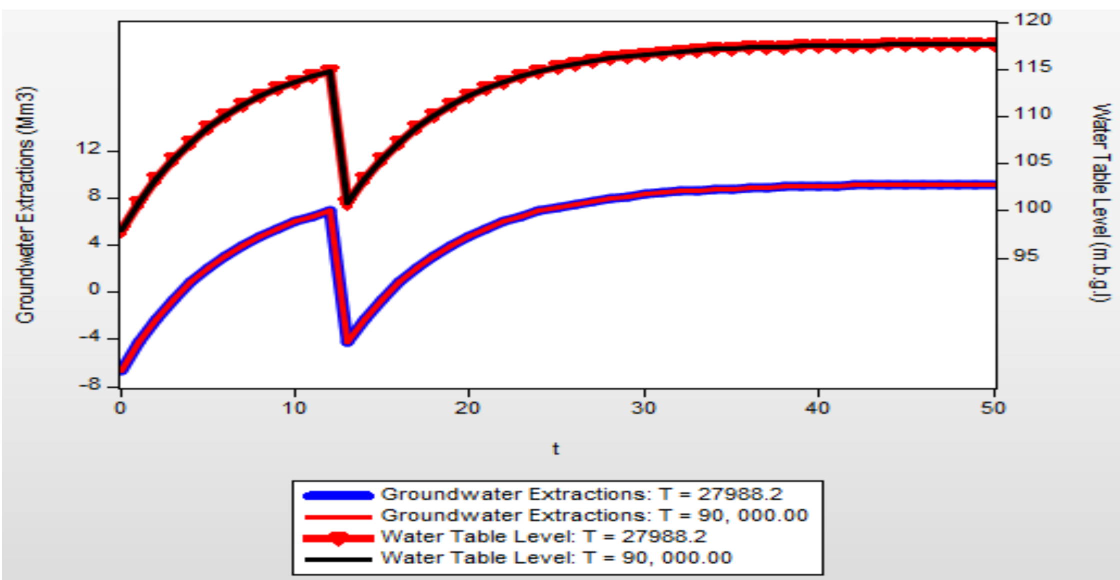

Figure 6: Dendron aquifer with taxes on LS and different values of the transmissivity.

Finally, the level of transmissivity or the radius of influence of the Dendron aquifer has no effect on groundwater extractions and the water table level (Figures 6 and 7). It's also worth noting that the private NPV is the same for all of the sensitivities examined here (Figure 8), thus there's no need for presenting a private NPV graph. Figure 8 illustrates that Farmers benefit from groundwater consumption when only the depth externality is considered (private NPV). When the two taxes on LS externalities are considered, the social welfare is negative. In addition, farmers' profits and social welfare levels are more reduced when the economic cost of land sinking is high and also when the transmissivity value is high.

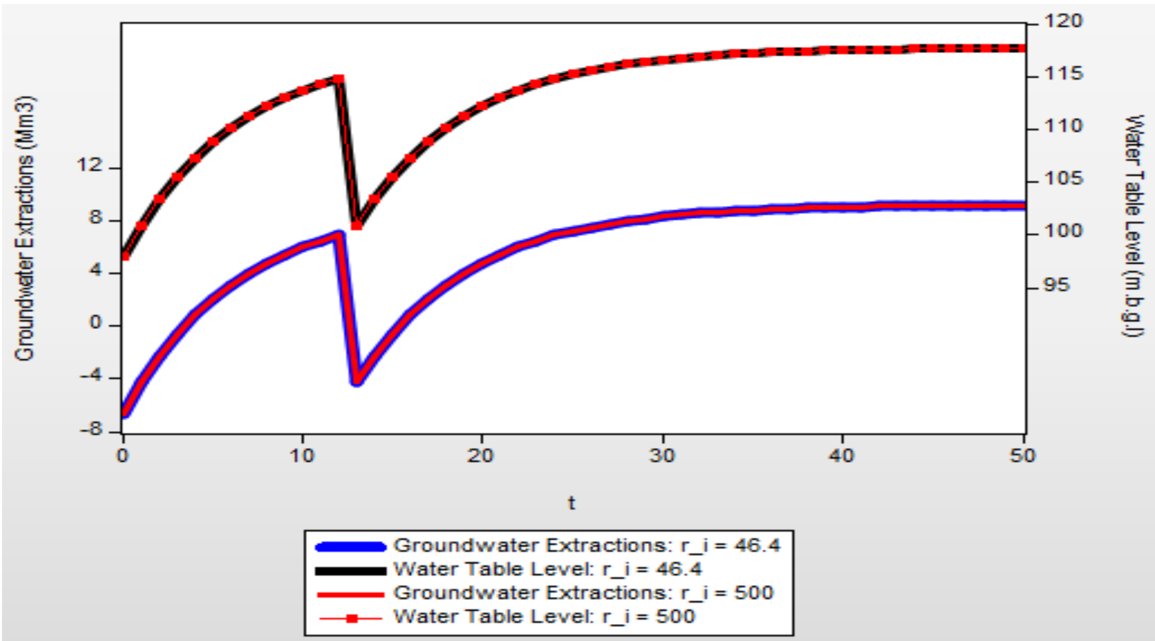

Figure 7: Dendron aquifer with taxes on LS and different values of the radius of influence.

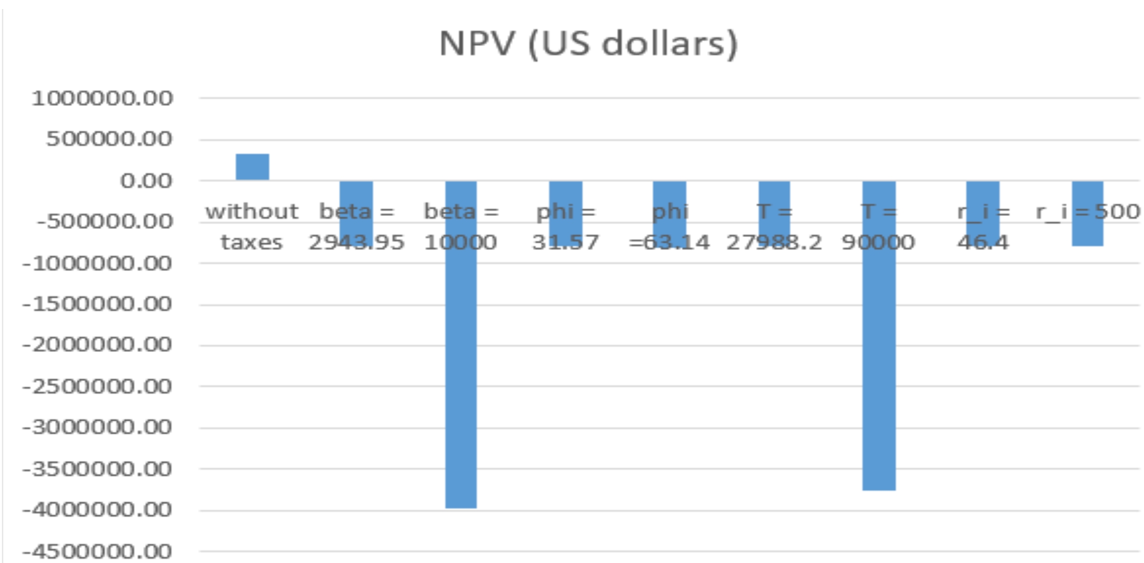

Figure 8: Social level NPV for the Dendron aquifer with taxes on LS (discount rate equal to 0.08).

We can conclude that alternative regulatory policies in the aquifer should be chosen to avert aquifer depletion and LS impacts while also optimising the social welfare because we found that this tax does not influence the water table level and extractions. Similarly, high tax values on the indirect LS negative externality result in lower social welfare, whereas low tax values result in higher social welfare. This tax delays aquifer depletion, but it does not benefit farmers or society. As a result, implementing it in the aquifer should be avoided to make way for other regulatory policies that may be more effective in averting LS impacts. As mentioned already, the NPV of social welfare is positive and very high (1,696,061,155,669,340.00 US dollars) in the hypothetical aquifer that is particularly prone to LS (unlike the Dendron aquifer, which is not prone to LS). This suggests that when an aquifer is prone to LS, the two taxes on LS externalities benefit farmers and society. As a result, such taxes should be imposed only on these aquifers to assist farmers financially and also reduce LS externalities that affect society.

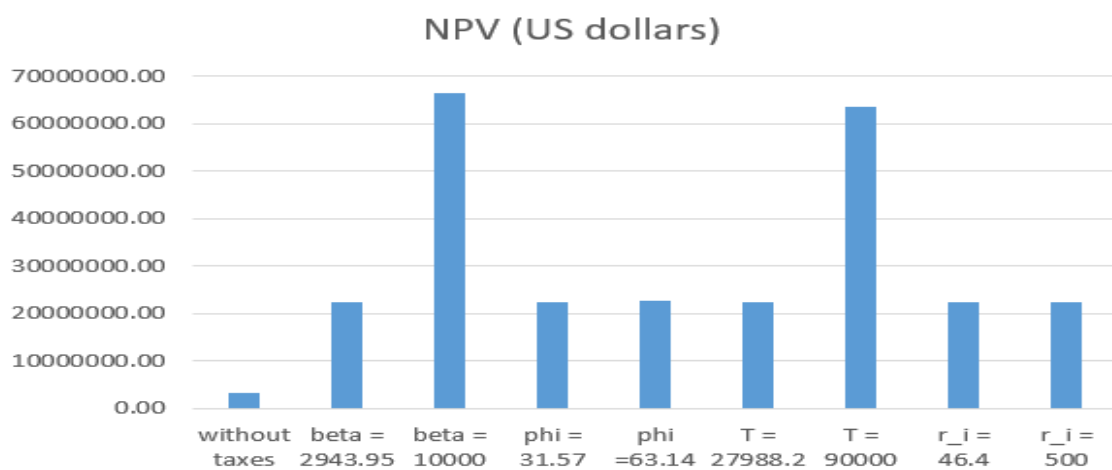

Figure 9: Social level NPV for the Dendron aquifer with taxes on LS when the social discount rate is 2.3%.

Farmers from the Dendron aquifer benefit from LS taxes when the social discount rate is low, as seen in Figure 9. We notice that all of the NPV values are positive, as opposed to being negative under the 8% social discount rate. When the economic cost of land sinking and the transmissivity values are high, society benefits more from these levies. It's also worth noting that the private NPV is the same for all of the sensitivities examined here, thus there's no need for a private NPV graph. Figure 10 demonstrates that when quotas are implemented, the steady-state for groundwater extractions is reached after nearly 40 years (compared to 33 years without policy intervention), whereas the water table level takes about 35 years (34 years when there is no policy intervention). The level of the quota has little effect on extractions and the level of the water table; the optimal paths stay the same. The main thing to note here is that the water table level falls dramatically compared to when LS taxes are considered, as well as when no policy is considered at all. In terms of water conservation, levies are therefore preferable to quotas.

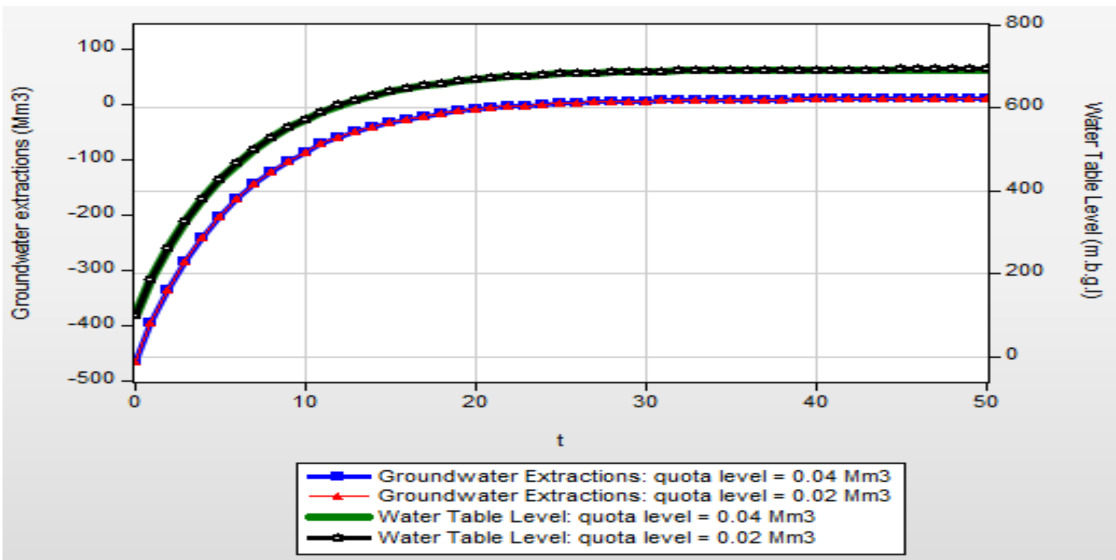

Figure 10: Dendron aquifer with different levels of quotas.

Society benefits financially from the establishment of quotas, as shown in Figure 11, because the NPV for quotas is substantially higher than when there is no policy at all (private NPV). Similarly, when the discount rate is low or the quota level is low, the social NPV is substantially higher. Figure 12 indicates that in the Dendron aquifer, quotas and taxes preserve the water table level between 117.4 and 117.8 m.b.g.l for over 50 years. In fewer than 30 years, both extractions and the water table level will have achieved a steady-state. Farmers lose 9,896,777 US dollars. However, the initial water table level has risen (from 98 to 117.4 m.b.g.l, contradicting the water table dynamics' initial requirements. As a result, taxes are more beneficial than quotas.

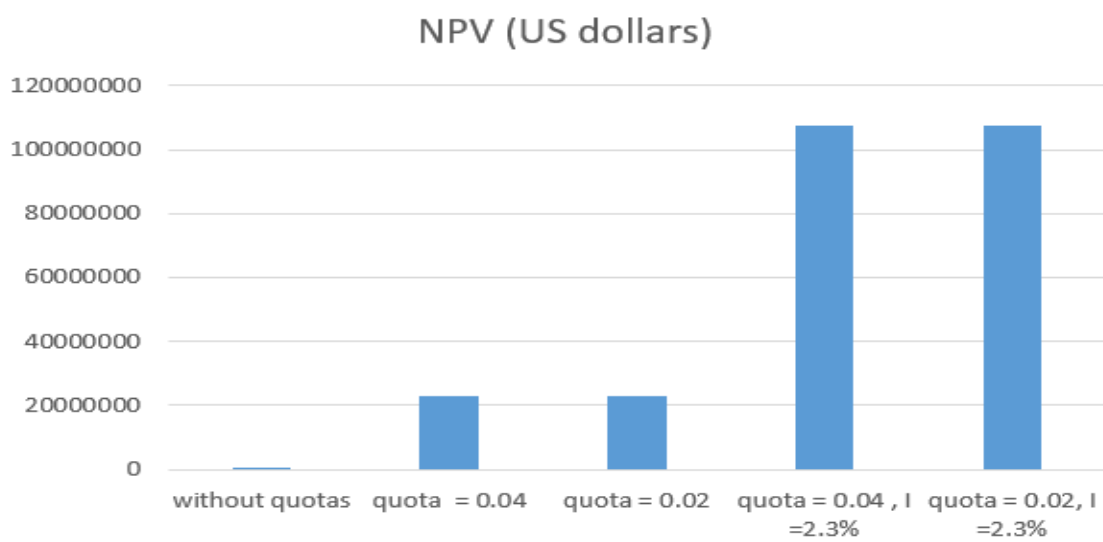

Figure 11: Social NPV for the Dendron aquifer with different levels of quotas and discount rates.

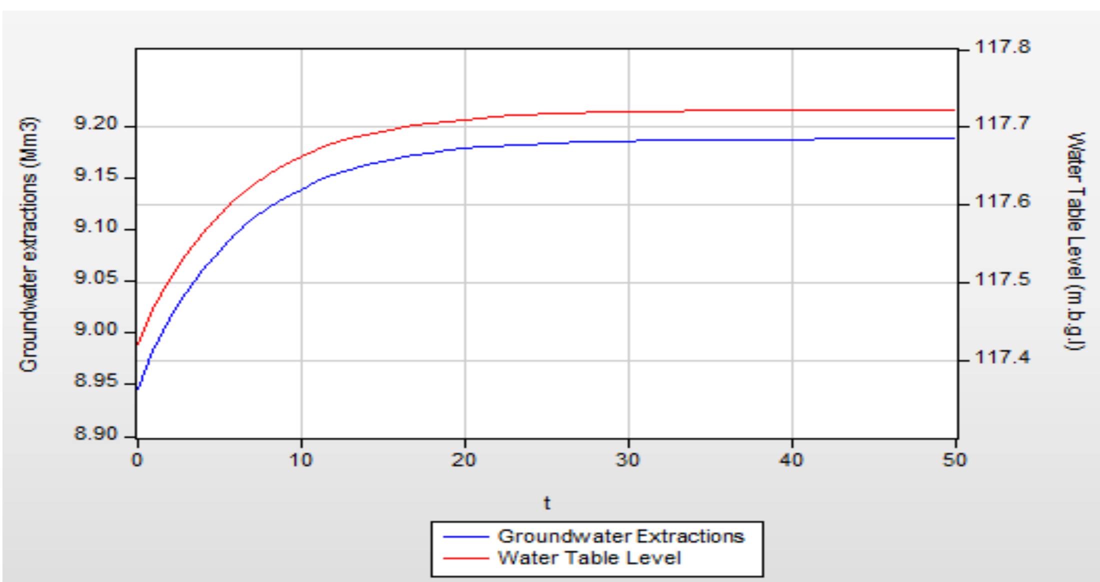

Figure 12. Dendron aquifer with both quotas and taxes on LS.

6. Conclusion and Policy Implications

Not only physically, but also economically, land subsidence poses a serious threat to people, infrastructure, and the environment. However, very little has been done to date in terms of the economic analyses of LS and its management. The theoretical and empirical application results presented have significant implications for managing LS and addressing its consequences. We found that, for effective and optimal groundwater management policies, a better understanding of the effects of groundwater pumping on land sinking and inelastic aquifer compaction is required. Most of the time, policymakers either attempt to mitigate the bad effects of LS without regard for the groundwater scarcity problem, or they attempt to solve the groundwater scarcity problem without regard for LS. However, addressing the two issues concurrently minimizes the amount of effort and resources spent on LS damages and water scarcity.

By using an economic optimization model of groundwater use, we were able to show that groundwater scarcity may be alleviated while also minimizing LS impacts. We discovered that understanding groundwater pumping effect on the direct LS damage externality is not required for optimal management of rock-based aquifer lithologies such as the Dentron aquifer. This is because these aquifers are less susceptible to LS, thus imposing direct LS negative externality policies on groundwater withdrawal does not alleviate either the LS problem or the water scarcity problem. However, successful groundwater management in sediment-based aquifer lithologies necessitates a thorough understanding of the direct (LS) damage externality. We discovered that the economic cost of land sinking and that of losing aquifer storage capacity have an effect on groundwater extractions and water table levels. The investigation of the indirect loss of aquifer storage capacity owing to LS is one of the study's significant findings.

To summarize, holding groundwater extractors liable for the Dendron aquifer's indirect loss of storage capacity delays the aquifer's depletion, but nothing can be done once the first delay occurs. We discovered that when there is no land subsidence and no regulatory policy in place, farmers benefit financially from groundwater in the Dendron aquifer. This profit is made, however, at the expense of draining the aquifer within a short period. This finding indicates that the aquifer is being over-exploited, necessitating prompt intervention. When there is land subsidence and still no regulatory policy, farmers and society face a loss. Farmers and society do not gain financially from the land subsidence levies proposed in the Dendron aquifer. However, the loss is made at the expense of sustaining the aquifer to avoid depletion within a short time.

We discovered that over the planning horizon, farmers and society benefit from groundwater in sediment-based lithologies aquifers. As a result, the imposed taxes benefit farmers and society financially.

We further conclude that implementing the two taxes on LS in the Dendron aquifer should be avoided to make way for other regulatory policies that may be more effective in averting LS impacts. High tax values on both the direct and the indirect LS negative externalities result in poor social welfare (huge losses), whereas low tax values result in better social welfare (small losses). When LS is considered, the farmers' social welfare is positive and very high in aquifers that are particularly prone to LS. As a result, such taxes should be imposed only in these aquifers to assist farmers financially and also reduce LS externalities. Farmers and society benefit from groundwater consumption when only the depth externality is considered. Even when the two taxes on LS externalities are taken into account, the farmer's private NPV is always positive. We also discovered that when the discount rate is smaller, farmers and society in the Dendron aquifer gain from the implementation of taxes and quotas regulatory policies. In comparison to the case when no policy intervention is made, farmers profit financially from the imposition of quotas. As a result, in terms of social welfare benefits, quotas are more favorable to farmers and society than taxes. Taxes are preferable to quotas when it comes to groundwater conservation.

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Appendix

Construction of the indirect-damage rate function

Pumping groundwater from aquifers compacts compressible fine-grained sediments within or next to aquifers (Leake and Prudic, 1991, p.1). This is mainly because coarse-grained sediments compaction may be reversed when groundwater is replenished, given the fact that they have no impact on the aquifer's storage capacity (Williamson et al., 1989, p.97). All aquifer systems, according to Holzer and Galloway (2005), compact to some degree in response to a change in groundwater level. Compaction is controlled, in theory, by effective stress (Holzer and Galloway, 2005). As suggested by Terzaghi (1925), effective stress is given by Equation (29) below

$$\delta' = \delta - \mu \tag{29}$$

where δ', δ, and μ represent the effective stress, the total pressure (geostatic stress), and the pore-fluid pressure (neutral or hydrostatic stress), respectively. Removing groundwater from sediments lowers the pore-fluid pressure within the sediments (Holzer and Galloway, 2005). As a result, the effective stress rises, and the pore space (or pore volume) decreases. This process is referred to by hydrologists as compaction (Poland et al., 1972). The change in effective stress has been shown to be proportional to the amount of compaction (Riley, 1969; Helm 1975; Leake and Prudic, 1991, p.3). The change in effective stress in an unconfined aquifer depends on the change in water table level (Leake and Prudic, 1991, p.3). Thus, we define the change in effective stress for an unconfined aquifer as suggested by Poland and Davis (1969, p.195)

$$\Delta\delta' = -\gamma_w(1 - n + n_w)\Delta H \tag{30}$$

where $\Delta\delta'$, γ_w, n, n_w, and ΔH represent the change in effective stress (positive for a rise and negative for a reduction), the unit weight of water (N/m^3), the porosity (dimensionless), the moisture content of sediments above water table (in the unsaturated zone) as a fraction of total volume (dimensionless), and the change in water table (positive for raising and negative for lowering). Here also, as under the direct LS externality, the only action of the pumper that affects the change in water table is withdrawals (W), as the natural recharge and percolation are not determined by the pumper but by some natural phenomenons, ΔH is considered to be negative throughout. This implies the lowering of the water table due to pumping, which is termed as drawdown $D(W)$. That is, we assume that the pumper is penalized for any of her action (in this case, simply withdrawals) that leads to inelastic compaction. As a result, Equation (30) becomes

$$\Delta\delta' = \gamma_w(1 - n + n_w)D(W). \tag{31}$$

Compaction, on the other hand, occurs whenever there is an increase in the effective stress. However, inelastic (permanent) compaction, which results in the loss of aquifer storage capacity, occurs only when the effective stress exceeds the pre-consolidation stress (Holzer and Galloway, 2005; Lofgren, 1975, p.40). Pre-consolidation stress refers to the highest effective stress that a soil has experienced over its life (Yang et al., 2009). Any rise in effective stress value lower than the pre-consolidation stress causes elastic compaction, in which sediment deformations can be reversed by replenishing the aquifer. When inelastic compaction occurs, pore space is permanently lost and cannot be restored. This means that the aquifer's storage capacity is lost forever. Even if the aquifer's water level is restored throughout, it will not be able to contain the same volume of water as it did before the compaction (Williamson et al., 1989, p.97). When the aquifer experiencing subsidence is replenished and then groundwater levels fall again, significant compaction will not resume until the new pre-consolidation stress is surpassed (Holzer and Galloway, 2005; Leake and Prudic, 1991, p.4). As suggested by Poland (1969, p.288-290), the approximate inelastic compaction Δq (in m) is given by Equation (32) below

$$\Delta q = y_v y \Delta\delta' \tag{32}$$

where y_v and y represent the compacting beds' mean compressibility and the aggregate thickness, respectively. As mentioned earlier, we assume a single-cell aquifer with a non-heterogeneous distribution of impacts and wells. Without loss of generality, we assume that the compacting beds' aggregate thickness is equal to the aquifer's thickness b.

Proof of Proposition (1)

The hamiltonian function of the system (15), (16), (17) is given as follows

$$\mathcal{H}_2(t, W_2, H_2, \lambda_2) = -\mathrm{e}^{-it}[\frac{W_2^2}{2k} - \frac{gW_2}{k} - (C_0 + C_1H_2)W_2 - G_2W_2]$$

$$+\lambda_2 \cdot \frac{[R+(\alpha-1)W_2]}{AS} \tag{33}$$

Where

$$G_2 = (\beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi + \phi \cdot b y_v \gamma_w (1-n+n_w)) \frac{1}{2\pi T} \ln(\frac{r_i}{r_w}). \tag{34}$$

Hence, the first order conditions are as follows

$$\frac{\partial \mathcal{H}_2}{\partial W_2} = -\mathrm{e}^{-it}[\frac{W_2}{k} - \frac{g}{k} - C_0 - C_1 H_2 - G_2] + \lambda_2[\frac{(\alpha-1)}{AS}] = 0. \tag{35}$$

$$\dot{\lambda}_2 = -\frac{\partial \mathcal{H}_2}{\partial H_2}. \tag{36}$$

$$\dot{H}_2 = \frac{1}{AS}[R + (\alpha-1)W_2]. \tag{37}$$

The transversality condition is given by $\lim_{t\to\infty} \lambda_2(t) = 0$. From Equation (35), we obtain the value for the costate variable λ_2 as follows.

$$\lambda_2 = \frac{1}{m}\mathrm{e}^{-it}[(\frac{1}{k})W_2 - \frac{g}{k} - C_0 - C_1 H_2 - G_2], \tag{38}$$

where $m = \frac{(\alpha-1)}{AS}$. The derivative of λ_2 with respect to t is given by

$$\dot{\lambda}_2 = \frac{1}{m}\mathrm{e}^{-it}[-\frac{iW_2}{k} + \frac{ig}{k} + iC_0 + iC_1 H_2 + iG_2 - \frac{C_1 R}{AS} - C_1 m W_2 + \frac{\dot{W}_2}{k}]. \tag{39}$$

The derivative of $\mathcal{H}_2$ with respect to the water table elevation H_2 is given by

$$-\frac{\partial \mathcal{H}_2}{\partial H_2} = -C_1 W_2 \mathrm{e}^{-it}. \tag{40}$$

From Equation (36), we obtain the following equation.

$$-C_1 W_2 = \frac{1}{m}\mathrm{e}^{-it}[-\frac{iW_2}{k} + \frac{ig}{k} + iC_0 + iC_1 H_2 + iG_2 - \frac{C_1 R}{AS} - C_1 m W_2$$
$$+\frac{\dot{W}_2}{k}]. \tag{41}$$

Solving for $\dot{W}_2$ in the above equation we get the following equations.

$$\frac{\dot{W}_2}{mk} = \frac{iW_2}{mk} - \frac{ig}{mk} - \frac{iC_0}{m} - \frac{iC_1 H_2}{m} - \frac{iG_2}{m} + \frac{C_1 R}{mAS} + \frac{C_1 m W_2}{m} - C_1 W_2 \tag{42}$$

$$\frac{\dot{W}_2}{k} = \frac{iW_2}{k} - \frac{ig}{k} - iC_0 - iC_1 H_2 - iG_2 + \frac{C_1 R}{AS} + C_1 m W_2 - C_1 m W_2 \tag{43}$$

$$\dot{W}_2 = iW_2 - ig - iC_0k - iC_1H_2k - kiG_2 + \frac{kC_1R}{AS} \tag{44}$$

$$\dot{W}_2 = iW_2 - ikC_1H_2 + [-ig - ikC_0 - ikG_2 + \frac{C_1kR}{AS}]. \tag{45}$$

Likewise, the value for $\dot{H}_2$ can be rewritten as

$$\dot{H}_2 = \frac{(\alpha-1)W_2}{AS} + \frac{R}{AS}. \tag{46}$$

Consequently, we now have to solve the two simultaneous differential equations ((45) and (46)). Thus, by letting $mm = \frac{(\alpha-1)}{AS}$, $uu = ikC_1$, $NN = -ig - ikC_0 - ikG_2 + \frac{C_1kR}{AS}$ and $MM = \frac{R}{AS}$, we get the following system of differential equations.

$$\dot{W}_2 = iW_2 - uu \cdot H_2 + NN. \tag{47}$$

$$\dot{H}_2 = mm \cdot W_2 + MM. \tag{48}$$

Putting the above system of differential equations in a D operator format (where $D = \frac{d}{dt}$), and solving for W_2 yields the following second order linear non-homogeneous differential equation.

$$[(D^2 - Di) + uu \cdot mm]W_2 = -uu \cdot MM. \tag{49}$$

The particular solution of the above differential equation is given by: $-\frac{MM}{mm}$ and the characteristic roots by $x_{1,2} = \frac{i \pm \sqrt{i^2 - 4uumm}}{2}$. Furthermore, the steady state level water table is given by

$$H_2^* = [\frac{-i\frac{MM}{mm} + NN}{uu}] \tag{50}$$

Hence, the solution for $W_2^*(t)$ is given by

$$W_2^*(t) = \frac{x_2}{mm}[H_T - \frac{NN - i\frac{MM}{mm}}{uu}]e^{x_2(t-t_T)} - \frac{MM}{mm}. \tag{51}$$

Where x_2 is the stable characteristic root. The negative root x_2 is the stable characteristic since we assume that all parameters are non negative, that is H_2^* and W_2^*, the steady state solutions. Likewise, the solution for $H_2^*(t)$ is given by

$$H_2^*(t) = [H_T - \frac{NN - i\frac{MM}{mm}}{uu}]e^{x_2(t-t_T)} + \frac{NN - i\frac{MM}{mm}}{uu}. \tag{52}$$

Proof of Proposition (4)

We can now solve the first sub-problem since we have the solution (SP_2^*) to the second sub problem. The hamiltonian function of the system (22), (23), (24) is given as follows

$$\mathcal{H}_1(t, W_1, H_1, \lambda_2) = -\mathrm{e}^{-it}[\frac{W_1^2}{2k} - \frac{gW_1}{k} - (C_0 + C_1H_1)W_1 - G_1W_1]$$
$$+\lambda_1 \cdot \frac{[R+(\alpha-1)W_1]}{AS} \tag{53}$$

Where

$$G_1 = \beta \cdot \eta \cdot \varepsilon \cdot b \cdot \psi \frac{1}{2\pi T} \ln(\frac{r_i}{r_w}). \tag{54}$$

Hence, the first order conditions are as follows

$$\frac{\partial \mathcal{H}_1}{\partial W_1} = -\mathrm{e}^{-it}[\frac{W_1}{k} - \frac{g}{k} - C_0 - C_1H_1 - G_1] + \lambda_1[\frac{(\alpha-1)}{AS}] = 0. \tag{55}$$

$$\dot{\lambda}_1 = -\frac{\partial \mathcal{H}_1}{\partial H_1}. \tag{56}$$

$$\dot{H}_1 = \frac{1}{AS}[R + (\alpha - 1)W_1]. \tag{57}$$

The transversality condition is given by $\lim_{t\to\infty}\lambda_1(t) = 0$. From Equation (55), we obtain the value for the costate variable λ_1 as follows.

$$\lambda_1 = \frac{1}{m}\mathrm{e}^{-it}[(\frac{1}{k})W_1 - \frac{g}{k} - C_0 - C_1H_1 - G_1], \tag{58}$$

where $m = \frac{(\alpha-1)}{AS}$. The derivative of λ_1 with respect to t is given by

$$\dot{\lambda}_1 = \frac{1}{m}\mathrm{e}^{-it}[-\frac{iW_1}{k} + \frac{ig}{k} + iC_0 + iC_1H_1 + iG_1 - \frac{C_1R}{AS} - C_1mW_1 + \frac{\dot{W}_1}{k}]. \tag{59}$$

The derivative of $\mathcal{H}_1$ with respect to the water table elevation H_1 is given by

$$-\frac{\partial \mathcal{H}_1}{\partial H_1} = -C_1W_1\mathrm{e}^{-it}. \tag{60}$$

From Equation (56), we obtain the following equation.

$$-C_1W_1 = \frac{1}{m}\mathrm{e}^{-it}[-\frac{iW_1}{k} + \frac{ig}{k} + iC_0 + iC_1H_1 + iG_1 - \frac{C_1R}{AS} - C_1mW_1$$

$$+\frac{\dot{W}_1}{k}]. \tag{61}$$

Solving for $\dot{W}_1$ in the above equation we get the following equations.

$$\frac{\dot{W}_1}{mk} = \frac{iW_1}{mk} - \frac{ig}{mk} - \frac{iC_0}{m} - \frac{iC_1H_1}{m} - \frac{iG_1}{m} + \frac{C_1R}{mAS} + \frac{C_1mW_1}{m} - C_1W_1 \tag{62}$$

$$\frac{\dot{W}_1}{k} = \frac{iW_1}{k} - \frac{ig}{k} - iC_0 - iC_1H_1 - iG_1 + \frac{C_1R}{AS} + C_1mW_1 - C_1mW_1 \tag{63}$$

$$\dot{W}_1 = iW_1 - ig - iC_0k - iC_1H_1k - kiG_1 + \frac{kC_1R}{AS} \tag{64}$$

$$\dot{W}_1 = iW_1 - ikC_1H_1 + [-ig - ikC_0 - ikG_1 + \frac{C_1kR}{AS}]. \tag{65}$$

Likewise, the value for $\dot{H}_1$ can be rewritten as

$$\dot{H}_1 = \frac{(\alpha-1)W_1}{AS} + \frac{R}{AS}. \tag{66}$$

Consequently, we now have to solve the two simultaneous differential equations ((65) and (66)). Thus, by letting $m = \frac{(\alpha-1)}{AS}$, $u = ikC_1$, $N = -ig - ikC_0 - ikG_1 + \frac{C_1kR}{AS}$ and $M = \frac{R}{AS}$, we get the following system of differential equations.

$$\dot{W}_1 = iW_1 - u \cdot H_1 + N. \tag{67}$$

$$\dot{H}_1 = m \cdot W_1 + M. \tag{68}$$

Putting the above system of differential equations in a D operator format (where $D = \frac{d}{dt}$), and solving for W_1 yields the following second order linear non-homogeneous differential equation.

$$[(D^2 - Di) + u \cdot m]W_1 = -u \cdot M. \tag{69}$$

The particular solution of the above differential equation is given by: $-\frac{M}{m}$ and the characteristic roots by $y_{1,2} = \frac{i \pm \sqrt{i^2 - 4um}}{2}$. Furthermore, the steady state level water table is given by

$$H_1^* = [\frac{-i\frac{M}{m} + N}{u}] \tag{70}$$

Hence, the solution for $W_1^*(t)$ and $H_1^*(t)$ is given by

$$W_1^*(t) = \tilde{A}\mathrm{e}^{y_1 t} + \tilde{B}\mathrm{e}^{y_2 t} - \frac{M}{m}. \tag{71}$$

$$H_1^*(t) = \frac{m}{y_1}\tilde{A}\mathrm{e}^{y_1 t} + \frac{m}{y_2}\tilde{B}\mathrm{e}^{y_2 t} + \frac{N-i\frac{M}{m}}{u}. \tag{72}$$

Where $\tilde{A}$ and $\tilde{B}$ are obtained by imposing the initial conditions.

$$\tilde{A} = \frac{y_1}{m}[H_0 - \frac{N-\frac{iM}{m}}{u} - \frac{[H_0-\frac{N-\frac{iM}{m}}{u}]\mathrm{e}^{y_1 t_T} - [H_T - \frac{N-\frac{iM}{m}}{u}]}{\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T}}]. \tag{73}$$

$$\tilde{B} = \frac{y_2}{m}[\frac{[H_0-\frac{N-\frac{iM}{m}}{u}]\mathrm{e}^{y_1 t_T} - [H_T - \frac{N-\frac{iM}{m}}{u}]}{\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T}}]. \tag{74}$$

Proof of Proposition (2)

To determine the impact of land sinking on the optimal solutions, we differentiate the expressions for the water table and extractions with respect to the economic cost of land sinking.

$$\frac{\partial W(t)}{\partial \beta} = \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T C_1} \ln(\frac{r_i}{r_w}) \cdot \frac{x_2 AS}{\alpha - 1} \mathrm{e}^{x_2(t-t_T)}. \tag{75}$$

We know that $\eta > 0$, $b > 0$, $T > 0$, $\mathrm{e}^{x_2(t-t_T)} > 0$, $\psi > 0$, $k < 0$, $C_1 < 0$, $(\alpha - 1) < 0$, and $\varepsilon > 0$ since an increase in the confining unit material or a compacting sediment induces a reduction in it's volume. If there was no x_2, the derivative's sign would be positive. Therefore, the sign of the derivative depends on the value of x_2. If $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the sign of the derivative is negative. If $i > \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the sign of the derivative is positive.

$$\frac{\partial H(t)}{\partial \beta} = \eta \cdot \varepsilon \cdot b \cdot \psi \cdot \frac{1}{2\pi T C_1} \ln(\frac{r_i}{r_w}) \cdot [\mathrm{e}^{x_2(t-t_T)} - 1]. \tag{76}$$

In this case, if there was no $(\mathrm{e}^{x_2(t-t_T)} - 1)$, the derivative's sign would be negative. Therefore, the sign of the derivative depends on the value of $(\mathrm{e}^{x_2(t-t_T)} - 1)$. If $\mathrm{e}^{x_2(t-t_T)} > 1$, the sign of the derivative is negative. If $\mathrm{e}^{x_2(t-t_T)} < 1$, the sign of the derivative is positive.

Proof of Proposition (3)

To determine the impact of the aquifer storage capacity reduction on the optimal solutions, we differentiate the expressions for the water table and extractions with respect to the economic cost of losing the aquifer's storage capacity.

$$\frac{\partial W(t)}{\partial \phi} = by_v\gamma_w(1-n+n_w)\frac{1}{2\pi TC_1}\ln(\frac{r_i}{r_w})\cdot\frac{x_2 AS}{\alpha-1}\mathrm{e}^{x_2(t-t_T)}. \quad (77)$$

We know that $y_v > 0$, $n > 0$, $n_w > 0$, $b > 0$, $T > 0$, $\mathrm{e}^{x_2(t-t_T)} > 0$, $\gamma_w > 0$, $k < 0$, $C_1 < 0$, $(\alpha - 1) < 0$, and $\gamma_w(1-n+n_w)\frac{1}{2\pi T}\ln(\frac{r_i}{r_w}) > 0$ according to Equation (31). If there was no x_2, the derivative's sign would be positive. Therefore, the sign of the derivative depends on the value of x_2. If $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the sign of the derivative is negative. If $i > \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$, the sign of the derivative is positive.

$$\frac{\partial H(t)}{\partial \phi} = by_v\gamma_w(1-n+n_w)\frac{1}{2\pi TC_1}\ln(\frac{r_i}{r_w})\cdot[\mathrm{e}^{x_2(t-t_T)}-1]. \quad (78)$$

In this case, if there was no $(\mathrm{e}^{x_2(t-t_T)}-1)$, the derivative's sign would be negative. Therefore, the sign of the derivative depends on the value of $(\mathrm{e}^{x_2(t-t_T)}-1)$. If $\mathrm{e}^{x_2(t-t_T)} > 1$, the sign of the derivative is negative. If $\mathrm{e}^{x_2(t-t_T)} < 1$, the sign of the derivative is positive.

Proof of Proposition (5)

To determine the impact of land sinking on the optimal solutions, we differentiate the expressions for the water table and extractions with respect to the economic cost of land sinking.

$$\frac{\partial W(t)}{\partial \beta} = \eta\cdot\varepsilon\cdot b\cdot\psi\cdot\frac{1}{2\pi TC_1}\ln(\frac{r_i}{r_w})\cdot\frac{(\alpha-1)y_2}{AS(\mathrm{e}^{y_1 t_T}-\mathrm{e}^{y_2 t_T})}\mathrm{e}^{y_2 t_T + y_2 t}. \quad (79)$$

We know that $\eta > 0$, $b > 0$, $T > 0$, $\psi > 0$, $k < 0$, $C_1 < 0$, $(\alpha - 1) < 0$, and $\varepsilon > 0$ since an increase in the confining unit material or a compacting sediment induces a reduction in it's volume. If there was no y_2 and $(\mathrm{e}^{y_1 t_T}-\mathrm{e}^{y_2 t_T})$, the derivative's sign would be positive. Therefore, the sign of the derivative depends on the value of y_2 and $(\mathrm{e}^{y_1 t_T}-\mathrm{e}^{y_2 t_T})$. If $i < \sqrt{i^2 - \frac{ikC_1(\alpha-1)}{AS}}$ and $(\mathrm{e}^{y_1 t_T} < \mathrm{e}^{y_2 t_T})$, the sign of the derivative is positive, otherwise the sign is negative.

$$\frac{\partial H(t)}{\partial \beta} = \eta\cdot\varepsilon\cdot b\cdot\psi\cdot\frac{1}{2\pi TC_1}\ln(\frac{r_i}{r_w})\cdot[\frac{\mathrm{e}^{y_2 t_T + y_2 t}(\alpha-1)^2}{(AS)^2(\mathrm{e}^{y_1 t_T}-\mathrm{e}^{y_2 t_T})}-1]. \quad (80)$$

In this case, if there was no $(\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T})$, the derivative's sign would be negative. Therefore, the sign of the derivative depends on the value of $(\mathrm{e}^{y_1 t_T} - \mathrm{e}^{y_2 t_T})$. If $\mathrm{e}^{y_1 t_T} > \mathrm{e}^{y_2 t_T}$, the sign of the derivative is negative. If $\mathrm{e}^{y_1 t_T} < \mathrm{e}^{y_2 t_T}$, the sign of the derivative is positive.

Proof of Proposition (6)

Using equation (18), we determine the value of t_T that satisfies the condition $W^{\star}(t) \leq \widehat{W}(t)$.

$$\frac{x_2 AS}{\alpha-1}\left[H_0 - \frac{NN - i\frac{R}{\alpha-1}}{ikC_1}\right]\mathrm{e}^{x_2(t-t_T)} - \frac{R}{\alpha-1} \leq \widehat{W} \tag{81}$$

$$\frac{x_2 AS}{\alpha-1}\left[H_0 - \frac{NN - i\frac{R}{\alpha-1}}{ikC_1}\right]\mathrm{e}^{x_2(t-t_T)} \leq \frac{\widehat{W}(\alpha-1)+R}{\alpha-1} \tag{82}$$

$$\left[H_0 - \frac{NN - i\frac{R}{\alpha-1}}{ikC_1}\right]\mathrm{e}^{x_2(t-t_T)} \leq \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \tag{83}$$

$$\left[\frac{H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR}{(\alpha-1)ikC_1}\right]\mathrm{e}^{x_2(t-t_T)} \leq \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \tag{84}$$

$$\mathrm{e}^{x_2(t-t_T)} \leq \frac{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)}{[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]x_2 AS} \tag{85}$$

$$\frac{\mathrm{e}^{x_2 t} x_2 AS[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]}{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)} \leq \mathrm{e}^{x_2 t_T} \tag{86}$$

$$\ln[\frac{e^{x_2 t} x_2 AS[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]}{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)}] \leq x_2 t_T \tag{87}$$

$$\frac{1}{x_2}\ln[\frac{e^{x_2 t} x_2 AS[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]}{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)}] \leq t_T \tag{88}$$

Since t_T is a constant throughout the planning horizon, we choose $t = 0$ to get the value

$$\frac{1}{x_2}\ln[\frac{x_2 AS[H_0(\alpha-1)ikC_1 - NN(\alpha-1) - iR]}{[\widehat{W}(\alpha-1)+R]ikC_1(\alpha-1)}] \leq t_T. \tag{89}$$

Proof of Proposition (7)

When both the economic costs attached to mitigating land subsidence impacts are equal to zero, the optimal path for groundwater extractions is given by

$$W^\star(t) = \frac{x_2 AS}{\alpha-1}[H_0 - \frac{N_0 - i\frac{R}{\alpha-1}}{ikC_1}]e^{x_2 t} - \frac{R}{\alpha-1}, \tag{90}$$

Where $N_0 = \frac{kC_1 R}{AS} - ig - ikC_0$. Using equation (90), we determine the value of N_0 that satisfies the condition $W^\star(t) \leq \widehat{W}(t)$.

$$\frac{x_2 AS}{\alpha-1}[H_0 - \frac{N_0 - i\frac{R}{\alpha-1}}{ikC_1}]e^{x_2 t} - \frac{R}{\alpha-1} \leq \widehat{W} \tag{91}$$

$$\frac{x_2 AS}{\alpha-1}[H_0 - \frac{N_0 - i\frac{R}{\alpha-1}}{ikC_1}]e^{x_2 t} \leq \frac{\widehat{W}(\alpha-1)+R}{\alpha-1} \tag{92}$$

$$[H_0 - \frac{N_0 - i\frac{R}{\alpha-1}}{ikC_1}]e^{x_2 t} \leq \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \tag{93}$$

$$[H_0 - \frac{N_0 - i\frac{R}{\alpha-1}}{ikC_1}] \leq \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \mathrm{e}^{-x_2 t} \tag{94}$$

$$H_0 - \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \mathrm{e}^{-x_2 t} \cdot ikC_1 \leq N_0 - \frac{iR}{\alpha-1} \tag{95}$$

$$H_0 - \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \mathrm{e}^{-x_2 t} \cdot ikC_1 + \frac{iR}{\alpha-1} \leq N_0 \tag{96}$$

Since N_0 is a constant throughout the planning horizon, we choose $t = 0$ to get the value

$$H_0 - \frac{\widehat{W}(\alpha-1)+R}{x_2 AS} \cdot ikC_1 + \frac{iR}{\alpha-1} \leq N_0. \tag{97}$$