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Designing optimal rainfall index-based crop production insurance: Application to the Limpopo River Basin of South Africa

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Summary:

The Rainfall-Index-Based Insurance (RIBI) is one type of crop insurance that is frequently used to mitigate the negative consequences of drought. This paper investigates the viability of a rainfall index insurance contract. We developed a single crop farm-level optimization model and solve it for various scenarios of rainfall distribution during the growing season. The model is tested by making use of daily rainfall data (1989-2020) from the Luvuvhu catchment of the Limpopo River Basin in South Africa. Using both grid rainfall data and weather station data, allows to estimate the value of a more precise rainfall data source. We looked at a RIBI that is based not only on the seasonal amount of precipitation, but also on sub-seasonal precipitation distribution. Grid data was found to be more effective than station data at reducing basis risk. For maize farmers in this area, the most beneficial policy is one in which crops are insured according to growth stages and the index is generated using grid data. In comparison to the uninsured, the insured has a lower expected utility. When compared to weather station data, grid data leads to lower premiums.

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Designing optimal rainfall index-based crop production insurance: Application to the Limpopo River Basin of South Africa

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Abstract

Rainfall-Index-Based Insurance (RIBI) is one type of crop insurance that is frequently used to mitigate the negative consequences of drought. This paper investigates the viability of a rainfall index insurance contract. We developed a single crop farm-level optimization model and solve it for various scenarios of rainfall distribution during the growing season. The model is tested by making use of daily rainfall data (1989-2020) from the Luvuvhu catchment of the Limpopo River Basin in South Africa. Using both grid rainfall data and weather station data allows us to estimate the value of a more precise rainfall data source. We looked at an RIBI that is based not only on the seasonal amount of precipitation but also on sub-seasonal precipitation distribution. Grid data was found to be more effective than station data at reducing basis risk. For maize farmers in this area, the most beneficial policy is one in which crops are insured according to growth stages and the index is generated using grid data. In comparison to the uninsured, the insured has a lower expected utility. When compared to weather station data, grid data leads to lower premiums.

Keywords: RIBI, Limpopo River Basin, Drought, Basis risk, Water scarcity.

1. Introduction

One of the most urgent challenges faced by many farmers and the agricultural sector around the world is extreme weather events and climate disasters such as droughts. South Africa is not an exception since the country has been negatively affected by several drought events over the last three decades. For instance, several drought episodes (during the years 1991/92, 1997/98, 2001/02, 2003/04, 2015/16, and 2017/18) have been experienced in the country. These have not only reduced the agricultural performance but also negatively affected the country's long-term objective of offering massive agricultural-driven employment opportunities, particularly to the previously disadvantaged group of its population (Austin, 2008; Bahta et al., 2016; Vogel et al., 2000; Mpandeli et al., 2015). Around 245,000 South Africans in the agricultural sector experienced a decrease in their livelihoods due to the 1991/92 drought event (Vogel et al., 2000), while the overall country area used for maize production dropped from over 5 million hectares (1986/87) to 2.7 million hectares in 2008 (Austin, 2008). Other negative implications associated with drought events include the loss of approximately 10 million South African rands countrywide, and the reduction in agricultural production of 8.4 percent (Bahta et al., 2016). This decrease has more severely affected the small-holder farmers who are more vulnerable to weather changes, given their limited technological and organizational capacities.

Although actions that aim at mitigating droughts can take several forms (i.e adoption of more efficient irrigation technologies, implementation of inter and multi-cropping systems, adoption of drought-tolerant crops, etc.), one approach that is often explored in recent years is the adoption of rainfall-index-based insurance (RIBI) (Hellmuth et al., 2009, pp.11-12; Woodard and Garcia, 2008; Vroege et al., 2019; Kath et al., 2018; Kost et al., 2012). RIBI refers to a crop insurance contract whose payments depend upon the performance of a rainfall index measured by a rain gauge (or other data sources like satellite-based data) at a predefined location during a certain period (Kost et al., 2012; Cole et al., 2011). The principle is to allow farmers to purchase insurance from a company to reduce the risks that are associated with drought events, which affect crop yields and agricultural performance. The scheme's primary objective is to build the resilience capacity of the farming systems by allowing farmers to transfer the risks associated with drought to the insurer. However, one must highlight that the scheme is often faced with some technical and informational challenges which limit its widespread use in certain regions of the world. Payment is not directly linked to the actual losses incurred by the farmer. This is because there might be situations when farmers experience losses that are due to other causes but drought. This is referred to as basis risk (Woodard and Garcia, 2008)[1]. The direct implications of such situations are that farmers may experience losses and not receive payments, or not experience losses but still receive compensations. As a result, premiums may be collected from farmers who have suffered losses, but who do not receive payments simply because of the lack of triggering of the rainfall index.

Conradt et al. (2015) found that basis risk diminishes when there is a strong correlation between the selected climatic factor index (i.e drought) and farm yield. One way to overcome the problem of basis risks is to make use of satellite-based rainfall data instead of the observational ground data when it comes to deciding on whether and how much to compensate the farmer. For some areas that exhibit limited observational ground data, this alternative rainfall measurement method is shown to yield more accurate estimates (Black et al., 2015). Another method to reduce basis risk is the use of synthetic weather that is constructed using several data sources to backfill missing gaps (Brissette et al., 2007; Dalhaus and Finger, 2016). Despite such limitations, however, RIBI seems to offer several benefits when it comes to mitigating the effects associated with drought events. The scheme eliminates the problems of moral hazard and adverse selection, encountered under conventional insurance policy schemes (Vroege et al., 2019; Kath et al, 2018).

We investigate the policy of index-based crop insurance in South Africa, one of the most water-stressed countries in the world. This is done by developing a microeconomic-based farm optimization model that investigates the optimal premium farmers are willing and able to pay to purchase RIBI and mitigate the effects of water scarcity when they expect future drought events. We determine the insurance payment that maximizes the expected utility of the farmer's terminal wealth, given the realized level of rainfall during the crop growing season. The model is tested by making use of data from the Luvuvhu catchment of the Limpopo river basin (LRB) in

[1] Basis risk refers to the risk that the payout of a RIBI doesn't correspond to actual farm yield losses. For instance, Mexican farmers received payments when they didn't experience losses in 2005, while in 2006, they experienced losses but not received payments (Scamilla et al., 2009, p.50). This risk is faced by both farmers and the insurers. Farmers (or insurers) face basis risk when actual farm yield losses are higher (lower) than the insurance payout. Most often, actual yield losses and the insurance payout are never equal since the yield losses depend on other factors that are not correlated with rain (i.e. temperature, pests and diseases, etc.).

South Africa. Among all the existing international river basins in Southern Africa, the LRB has been classified as the one that faces the worse water scarcity challenges. The particularity of the model developed in this paper is that it caters for seasonal as well as inter-annual drought variations that affect crop growth, depending on the specific stage of the crop growth process.

Most previous studies only looked at the conditions that drive the adoption of index-based crop insurance when drought is captured as an average event. We contribute to the stock of the existing literature by constructing and applying index-based crop insurance that is based not only on the seasonal amount of precipitation compared to the crop requirement but also on sub-seasonal precipitation distribution. This allows us to go beyond the conventional approach and take into account the different stages of crop development and the relative sensitivity of the overall crop (yield) to the stress days in each of the growing stages (Dalhaus and Finger, 2016; Bucheli et al., 2020). Another extension of our model covers the comparison of the optimal premiums with rainfall data from a weather station that is located in the vicinity of the farm versus weather data that makes use of satellite. The objective of this comparison is to offer a more precise estimation of the optimal compensation that must be paid to farmers when they aim to mitigate drought events in South Africa.

This paper is organized as follows: Section 2 describes the single crop farm-level optimization model we developed to investigate the viability of a rainfall-index insurance contract. We present some propositions that provide a general solution to the model (Their proof appears in the Appendix). Furthermore, in section 3, we discuss the Luvuvhu catchment area in South Africa, where the model is empirically applied. In Section 4, we describe the data and data-collection process. The findings of the empirical application are then presented and discussed in Section 5. Then finally, in Section 6, we conclude and discuss policy implications.

2. The model

Our point of departure is a farmer who wants to mitigate the risk of drought facing her farm to avoid a decrease in crop yield, and ultimately welfare loss. This is done by purchasing rainfall-index-based insurance (RIBI) provided by an insurance company. Assume that the farmer purchases the insurance contract at the beginning of the emergence stage (stage VE in Figure 1), the planting time. We further assume that the insurer pays the compensation (if payment is triggered) only after the maturity stage of the crop production cycle (stage R6), which is at the end of the harvesting time.

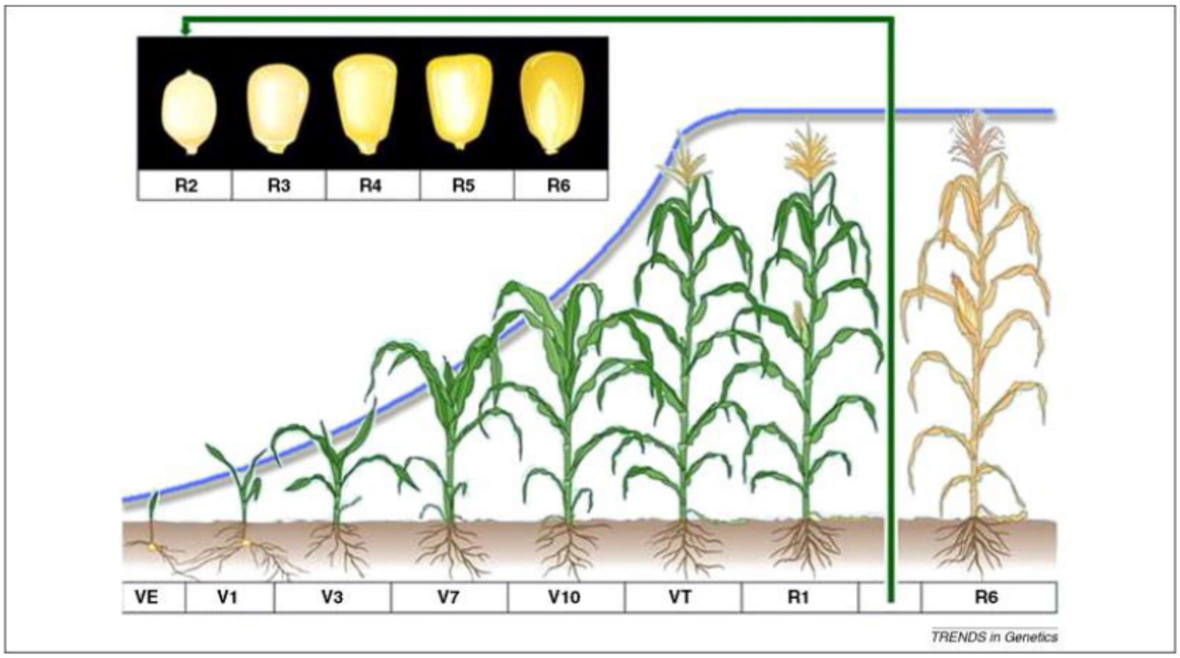

Figure 1: Maize crop growth stages.
Source: Dlamini (2015).

During the crop production cycle, the daily rainfall measurements are recorded. To avoid basis risk, we assume that drought is the dominant cause of crop losses in agricultural production at the vicinity of the farm, and that rainfall measurements are also carried out at the vicinity of the farm. In the absence of a weather station in the farm’s vicinity, the closest weather station from the surrounding areas is used. But, the shorter the distance between the weather station and the farm, the larger is the reduction of the basis risk. After the maturity stage (R6), the daily rainfall measurements are aggregated to form the total amount of rainfall received over the crop production cycle. The cumulative rainfall index which represents the total amount of rainfall received over the crop cycle is given by $X = \sum_{t=1}^{n} R_t$, where t and R represent the time in days and the observed rainfall measurement in mm, respectively. The insurance payment is triggered when the cumulative rainfall index (X) falls below the crop water requirement x_g. In this paper, the crop water requirement is referred to as the threshold or trigger value. The insurer and the farmer agree to the specific value of the threshold when they enter into the contractual agreement. The insurance payment is given by $h(X) = q \cdot max(x_g - X, 0)$, where q represents the conversion factor that transforms the rainfall index into money. In addition, the parameter q represents the insurer's monetary payment (to the farmer) per mm of rainfall below the crop water requirement. The distance between x_g and X provides the deviation of the cumulative rainfall value from the threshold. If that deviation is positive, compensation is paid to the farmer, and as highlighted above, regardless of whether the farmer has encountered losses or not. When the deviation is negative (or equal to zero), the farmer receives no compensation[2].

However, given the fact that there is always a high level of uncertainty associated with rainfall, we assume that X is a random variable that is defined over the set of rainfall climate in the vicinity of the farm $[0, \bar{x}]$. The set of possible rainfall values is defined as a collection of all values that X can take. The value $\bar{x}$ represents the maximum total rainfall that can be observed in the vicinity of the farm over the crop production cycle, the minimum being zero. The probability density function of X is given by $f(X) > 0$. We assume that $f(X)$ is continuous on the set of possible rainfall values $[0, \bar{x}]$. In the literature, precipitation variables have been modeled using several distributions. For instance, gamma, log-normal, Weibull, and the beta distributions have all been used to model the evolution of precipitation (Martin et al., 2001; Rey et al., 2016). Rainfall measurements cannot be negative, they only take positive values starting from (and including) zero to positive infinity. The set of possible rainfall values for which the aforementioned distributions are defined contains only non-negative values, which well coincide with all precipitation variables and indices. Except for the Weibull distribution, all distributions indicated above are not defined at $X = 0$. The beta distribution is defined at $X = 0$ but

[2] Since the premium is a function of x_g, the farmer has no incentive to declare a too high or too low values of x_g.

bounded above by $X = 1$, which does not correspond to the maximum total rainfall $\bar{x}$ that can be received in the vicinity of the farm. Therefore, we consider the Weibull distribution defined in Equation (1) below.

$$f(X) = \frac{m}{\psi}[\frac{X}{\psi}]^{m-1}\mathrm{e}^{-(X/\psi)^m},\ X \geq 0. \tag{1}$$

In equation (1), $m > 0$ and $\psi > 0$ represent the shape parameter and the scale parameter, respectively. An additional advantage of this distribution function is that its density can be truncated on the right (at $X = \bar{x}$), hence excluding the values that are not on the set of possible rainfall values of X. At the harvesting stage, the farmer harvests the remaining crop that survived the drought (if there was a drought) to generate revenue by selling on the market. The quantity produced by the farmer after the harvest is measured by a yield function. Most crop-water yield functions are quadratic in water because represent marginal damages to yield from over-irrigation in the form of soil aeration and salinity that lead to much of the over-irrigation not being efficiently used by the plant to crop production. Assuming a rational farmer, we employ a linear relationship that represents high application efficiency. We define the yield function $y(X)$ using the crop water production function as suggested by Conradt et al. (2015).

$$y(X) = c + \beta X + \varepsilon. \tag{2}$$

Where β, c, and ε, represent the slope coefficient (the change in yield per $1\ mm$ change in rainfall X), the intercept, and the random error, respectively. The random error accounts for other factors that are uncorrelated with rainfall but have an influence (or effect) on yield. In addition, the error term represents the unexplained variance[3] in yield. To determine the threshold, it is commonly assumed that the compensation to farmers is based on below-average rainfall amounts (Bucheli et al., 2020; Dalhaus and Finger, 2016; Conradt et al., 2015). The main assumption is that the average rainfall level gives (or leads to) the average crop yield. The index value that corresponds to $\bar{y}$ (the mean crop yield) becomes the threshold of the insurance contract. That is substitute $\bar{y}$, β, and c into the production function (2) and solve for the corresponding index value. Without loss of generality, we assume that rainfall is the only source

[3] The unexplained variance represents the basis risk of the RIBI (Conradt et al., 2015).

of water available for irrigation. The yield function is estimated using Quantile Regression (QR). Unlike the Ordinary Least Squares (OLS) which is a mean-conditioning approach, QR is a quantile-conditioning approach. The QR method minimizes the sum of absolute deviations (residuals not symmetrically weighted) instead of the sum of the squared deviations. The quantile of interest is specified by the parameter $0 < \tau < 1$. Lower values of τ imply that there is a lower chance that the observed yield is below the predicted yield, hence minimizing downside risk. The minimization problem is as follows

$$\hat{\beta}(\tau) = \text{argmin}_{\beta \in \mathbb{R}}[\tau \sum_{y \geq \beta X} |y - \beta X| + (1 - \tau) \sum_{y < \beta X} |y - \beta X|]. \quad (3)$$

Following Conradt et al. (2015) and Bucheli et al. (2020), we set $\tau = 0.3$ to sufficiently capture the low yield observations. We denote by z the premium paid by the farmer and assume that the premium is subsidized at a rate θ, with $0 \leq \theta < 1$. Therefore, assuming that the farmer's initial wealth is w (non-random), the farmer's expected utility of terminal wealth is given by

$$\mathbb{E}[U(w + p \cdot y(X) + h(X) - (1 - \theta)z)]. \quad (4)$$

Where U represents the utility of the farmer with the following characteristics: $U' > 0$ and $U'' \leq 0$. The market output (crop) price is denoted by p. We make use of a decreasing absolute risk aversion (DARA) utility function $U(Y) = \frac{Y^{1-r}}{1-r}, Y > 0, r \neq 1 \; and \; r > 0$. Previous studies have shown that most farmers exhibit primarily a decreasing absolute risk aversion (Rey et al., 2016). We further postulate that farmers are charged with a cost (premium) that is paid to receive insurance coverage. The insurance cost will be defined as the expected value of the insurance payment

$$z = \alpha \mathbb{E}[h(X)]. \quad (5)$$

Where $\alpha > 1$ is the insurance loading[4] ($\alpha = 1$ implies the actuarially fair insurance contract). Therefore, after combining (4) and (5) the maximization problem of the farmer becomes

[4] Insurance loading is an extra cost (which is part of an insurance contract) that covers losses which are higher than expected by the insurer. Such higher losses than anticipated arise from insured individuals who are susceptible (or prone) to a certain risk. For instance, the insured may be deemed prone to a certain risk due to her medical history (in medical insurance policies) or her past time exposure to an imminent risk of disaster (like farmers located in an area which is prone to droughts). The insurance loading also takes into account the insurer's transaction costs.

$$\max_{h(.)} \int_0^{\bar{x}} U[w + p \cdot y(X) + h(X) - (1-\theta)z]f(X)dX, \tag{6}$$

subject to

$$z = \alpha \int_0^{\bar{x}} h(X)f(X)dX, \tag{7}$$

$$h(X) \geq 0. \tag{8}$$

Equations (6) and (7) represent an isoperimetric problem in the optimization problem above. To solve the optimization problem, we convert the integral constraint into a differential equation, which transforms the entire problem into an optimal control problem. The aforementioned conversion is stated in the following theorem.

Theorem 1. (Equivalent constraint representation) *Let $h(X)$ and z be the payment function and the insurance premium, respectively. Assuming that z is fixed (constant), then the constraint (7) is equivalent to Equation (9) below*

$$\dot{s}(X) = \alpha h(X)f(X),\ s(0) = 0,\ s(\bar{x}) = z. \tag{9}$$

A proof of this theorem is given in the Appendix.

Therefore, the problem of the farmer is to determine the value of the insurance payment $h(X)$ that maximizes (6) subject to the constraints (8) and (9). Proposition 1 presents the optimal solution to this problem.

Proposition 1. (Optimal payment) *If the farmer maximizes terminal wealth according to (8 - 9), the optimal payment function ($h^*(X)$) is determined by the following expression.*

$$h^*(X) = \begin{cases} p[y(x_g) - y(X)] & \forall X \in [0, x_g), \\ 0 & \forall X \in [x_g, \bar{x}]. \end{cases} \tag{10}$$

A proof of this proposition is given in the Appendix.

Proposition 1 determines that the optimal payment is equivalent to the amount of money by which the actual farm yield's income falls short of the anticipated revenue when there is sufficient irrigation water. Therefore, rainfall index insurance compensates the farmer for the monetary losses experienced as a result of poor rainfall. Furthermore, we observe that the higher is the market output price, the higher is the optimal insurance payment. Low rainfall leads to low crop yield which further results in inflated prices of outputs. Prices are inflated to balance supply and demand as there will be a lower level of outputs to be supplied than what is demanded by the consumers. Hence, when output prices go up, the optimal insurance payment also goes up to fully compensate the farmer for the low crop yield obtained due to low rainfall. The limit of $\mathrm{h}^*(\mathrm{X})$, as X goes to zero, is equal to $\mathrm{p\beta x_g}$. The value $\mathrm{p\beta x_g}$ represents the maximum amount of money that the insurance company can pay to the farmer in the event that there was no rainfall recorded in the farm's vicinity throughout the growing season. Thus, the lower the total rainfall level on the farm throughout the contract (or the crop production cycle), the higher the payment from the insurer to the farmer, or the closer the payment (from the insurer to the farmer) to the value $\mathrm{p\beta x_g}$. Substituting the optimal payment from equation (10) into equation (7) produces the optimal premium that farmers are willing to pay to purchase the insurance contract. Proposition 2 presents the optimal premium.

Proposition 2. (Optimal premium) *The optimal cost* z^* *of the insurance policy is determined by the following expression.*

$$z^* = p\alpha\beta(x_g \cdot F(x_g) - \mathbb{E}[X_{[0,x_g)}]) \quad (11)$$

A proof of this proposition is given in the Appendix.

The cumulative density function $\mathrm{F(X)}$ is a right continuous, non-decreasing function. Hence, the premium rises as the crop's water requirement rise. The marginal product of water affects both the premium and the insurance payment. The higher the water's marginal product, the larger is the premium and the payoff to the farmer. We then proceed to the condition that must be met for the farmer to consider obtaining an index insurance contract or protection. The necessary condition for the farmer to purchase the insurance policy with the payoff $h^*(X)$ and premium z^* is

$$\mathbb{E}[U(w + p \cdot y(X) + h^*(X) - (1-\theta)z^*)] - \mathbb{E}[U(w + p \cdot y(X))] \geq 0. \tag{12}$$

In the empirical section of this paper, this condition will be examined. We further study the optimal payoff payment, when the distribution of rainfall is considered. This is because previous studies show that the distribution of rainfall could be more crucial in crop development than the overall amount of rainfall received over the crop growing cycle (Monti and Venturi, 2007). Crops require more water in some growth stages which are referred to as the critical stages. Once seeded, the seed soaks up water and begins to develop seedlings. If the soil is not sufficiently moist, the seed dies or either takes longer to germinate. This occurs mostly under rain-fed conditions where seeding is usually done when there is no adequate water in the soil, most often in dry soil while waiting for rainfall (Belfield and Brown, 2008). Emerging seedlings begin to provide water (and nutrients) to the crop from the soil which takes it to the second stage (stage V1 in Figure 1). Similarly, seedlings dry out under poor soil moisture conditions which causes the plant to die. Another critical period in the maize growth cycle is the silking stage (stage R1). During this stage, the crop requires a lot of water such that the silks don't dry out and fall off, which leads to fertilization, not occurring for all kernels, and hence a lower seed amount (Belfield and Brown, 2008; Hanway, 1966). Farmers' decisions to adopt an RIBI also vary based on the magnitude of the impact and risk at each stage of crop growth. For instance, at R1, there is a low risk attached since a larger portion of the growth cycle is observed, and there is more information on how the yield might turn out. But, a high-impact event could happen in the event of a drought. To incorporate the several growth stages of the crop, we define the yield function as: $y(X_i) =$

$c_i + \beta_i X_i + \varepsilon_i,\ 1 < i \leq m$. The subscript i represents the crop's growth stages. The farmer's expected utility of terminal wealth and her premium is given by $\mathbb{E}[U(w + p\sum_i y(X_i) + \sum_i h(X_i) - (1-\theta)z)]$ and $z = \alpha\mathbb{E}[\sum_i h(X_i)]$, respectively. Therefore, the maximization problem of the farmer becomes

$$\max_{h(.)} \int_0^{\bar{x}_i} U[w + p\sum_i y(X_i) + \sum_i h(X_i) - (1-\theta)z] f(X_i) dX_i, \tag{13}$$

subject to

$$z = \alpha \int_0^{\bar{x}_i} \sum_i h(X_i) f(X_i) dX_i, \tag{14}$$

$$\sum_i h(X_i) \geq 0. \tag{15}$$

In Proposition 1, we found the optimal payment to the farmer when crop growth stages are ignored but only the overall growing season is considered. Proposition 3 presents the optimal payment for the case where the growing season is sub-divided into multiple stages of crop growth. Likewise, in Proposition 2, we found the optimal premium for the case where crop growth stages are ignored. The optimal premium for the case where the growing season is sub-divided into multiple stages of crop growth is presented in Proposition 4.

Proposition 3. (Optimal payment) *If the farmer maximizes terminal wealth according to (14 - 15), the optimal payment function ($\sum_i h^*(X_i)$) is determined by the following expression.*

$$\sum_i h^*(X_i) = \begin{cases} p[\sum_i [y(x_{gi}) - y(X_i)]] & \forall\ X_i \in [0, x_{gi}), \\ 0 \quad \forall\ X_i \in [x_{gi}, \bar{x}_i]. \end{cases} \tag{16}$$

A proof of this proposition is given in the Appendix.

Proposition 4. (Optimal premium) *The optimal cost* z^* *of the insurance policy is determined by the following expression.*

$$z^* = p\alpha \sum_i \beta_i (x_{gi} \cdot F(x_{gi}) - \mathbb{E}[X^i_{[0,x_{gi})}]) \quad (17)$$

The proof of this proposition is similar to that of Proposition 2.

The total insurance payment to farmers over the growing season is simply the sum of the insurance payments made at each crop development stage. Likewise, the total premium to be paid by the farmer to the insurance company is the sum of the premiums quoted at each crop development stage. Proposition 3 demonstrates that the overall optimal payment over the growing season is equivalent to the sum of the amount of money by which the actual farm yield's income falls short of the anticipated revenue in the case that there is a sufficient amount of irrigation water at each crop growth stage. Furthermore, we observe that the higher the market output price, the higher the optimal insurance payment for individual crop growth stages and, as a result, the higher the overall growing season optimal insurance payment. The impact of the crop water requirement and the marginal product of water on the optimal payment and premium for individual growth stages is similar to the first scenario when growing stages were not taken into account. The only distinction is that some growth stages have a greater impact on the overall insurance payment and premium than others. For example, the higher the crop growth stage marginal product of water, the larger the premium and payment to the farmer during that growth stage. As a result, crop growth stages that are associated with a larger marginal product of water increase the farmer's overall payment compared to stages with a lower marginal product of water. The necessary condition for the farmer to purchase the insurance policy with the payoff $\sum_i h^*(X_i)$ and premium z^* when crop growth stages are considered is

$$\mathbb{E}[U(w + p\sum_i y(X_i) + \sum_i h^*(X_i) - (1-\theta)z^*)] - \mathbb{E}[U(w + p\sum_i y(X_i))] \geq 0. \quad (18)$$

The aforementioned results will be investigated in the empirical section of this study due to the complexity of the above expressions.

3. The study area: The Luvuvhu catchment of the Limpopo River Basin

The Luvuvhu catchment management area (Figure 2) is situated in the Limpopo Province of South Africa. The mean annual rainfall in the area ranges from 200 to 400 mm, with the rainfall season occurring between October to April (Singo et al., 2016). The catchment area remains under-developed and economically poor (Masupha et al., 2016). The total area covers around 5,941 square kilometers. According to Jewitt and Garratt (2004), subsistence agriculture and grazing take fifty percent of land use in the area. Maize is the main staple food in the catchment area, cultivated by small-scale subsistence farmers. A significant number of the area's inhabitants depend entirely on dry land maize cultivation, animal farming, as well as on remittances from employed family members who migrated to metropolitan areas townships (DWAF, 2004). Maize is completely rain-fed in the small-scale farming areas (Bouagila and Sushama, 2013), hence, it is affected by intense drought conditions due to the high rainfall variability in the area.

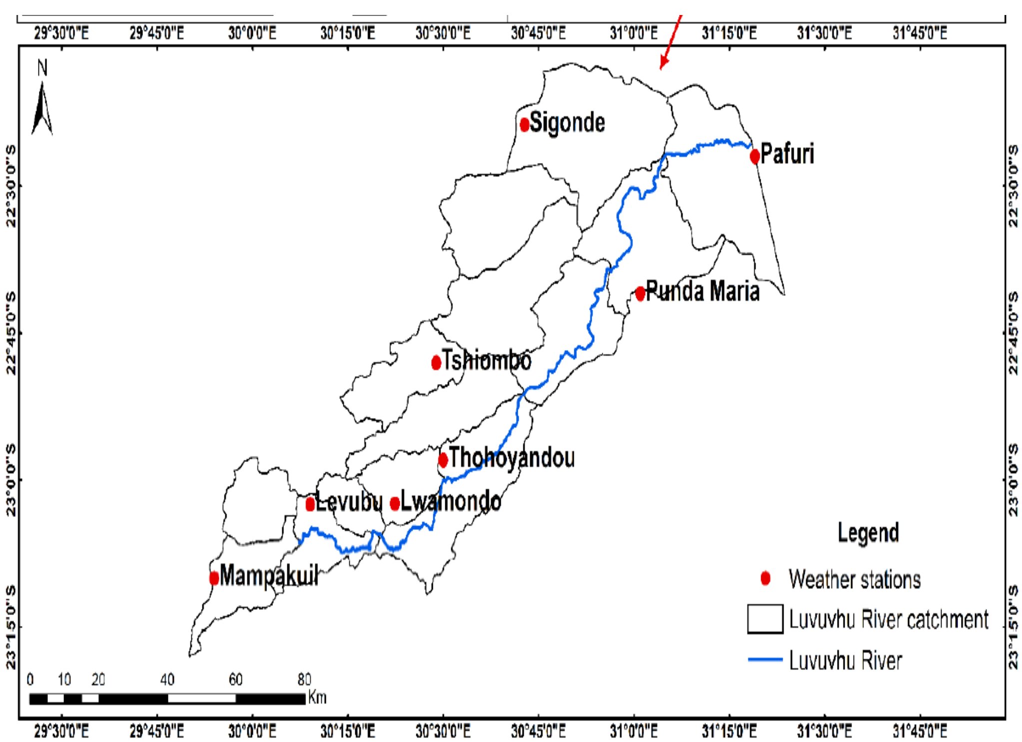

Figure 2: The Luvuvhu catchment area map.
Source: Masupha (2017).

4. Numerical applications

Daily precipitation and temperature data from the nearest meteorological station (from 1989 to 2020) used in this study were obtained from the South African Agricultural Research Council (ARC). The nearest weather station to the farm is the Levubu weather station. Missing rainfall data were estimated using the Inverse Distance Weighting (IDW) method which is used in previous studies (Bartier and Keller, 1996; Lu and Wong, 2008). The method assumes that the nearest weather stations have a huge influence on the target weather station data.

The gridded daily rainfall data was obtained from the Royal Netherlands Meteorological Institute (KNMI) Climate Explorer, and freely available online (https://climexp.knmi.nl/start.cgi). The KNMI Climate Explorer CPC (Climate Prediction Center) provides gridded daily precipitation data and long-term means of monthly and daily precipitation data. The data was produced by the NOAA Climate Prediction Center global unified gauge-based analysis of daily precipitation, with a dataset covering the period from 1979 to the present. It combines historical (and recent) land surface observations from different sources into global estimates of precipitation data using advanced data assimilation and forecasting models. The CPC Global Daily Unified Gauge-Based Analysis of Precipitation has a geospatial resolution of 0.5 degree latitude times 0.5-degree longitude.

Masupha et al. (2016) found that sowing maize in the Luvuvhu catchment area in October (early planting) exposed the crop to a higher risk of droughts than sowing it in December (late planting) or November (mid planting). Late planting exposes maize to frosts (as well as water shortages), slowing crop growth. Thus, in this study, planting is considered to begin in November each year. The growing period lasts from November to April, with winter frosts starting shortly before the end of April. The first day of the month is picked as the start date for the growing period. We follow previous studies and assume that the length of time it takes for a crop to attain maturity or finish a specific growth stage is directly related to temperature (Roberts et al., 2013), if we control for other inputs. Growing degree days (GDD) are used to calculate the end date of the growing period, as they determine when the crop enters each growth stage. Growing degree days were estimated mathematically using Equation (19) below (Anandhi, 2016)[5].

$$GDD = \frac{T_{max}+T_{min}}{2} - 10^{\circ}C. \quad (19)$$

$$T_{min} = \begin{cases} 30^{\circ}C & T_{min} \geq 30^{\circ}C, \\ T_{min} & 10^{\circ}C < T_{min} < 30^{\circ}C, \\ 10^{\circ}C & T_{min} \leq 10^{\circ}C. \end{cases} \quad (20)$$

$$T_{max} = \begin{cases} 30^{\circ}C & T_{max} \geq 30^{\circ}C, \\ T_{max} & 10^{\circ}C < T_{max} < 30^{\circ}C, \\ 10^{\circ}C & T_{max} \leq 10^{\circ}C. \end{cases} \quad (21)$$

The parameters T_{max} andT_{min} represent the daily maximum temperature and daily minimum temperature, respectively. A 1500 degree Celsius maize hybrid enters into the tassel initiation stage (V5) at 264 degrees Celsius, the tassel emergence stage (VT) at 639 degrees Celsius, the blister stage (R2) at 904 degrees Celsius, and the physiological maturity stage (R6) at 1500 degree Celsius (Neil and Newman,1990; Duncan et al., 2010). As a result, we divided the growing period into four stages: establishment (VE-V5), vegetative (V6-Vn)[6], flowering (VT-R1), and yield formation and ripening (R2-R5). Because harvesting occurs within the physiologic maturity stage, it is not included in the index calculation.

Given the fact that individual farm maize yield data from the Luvuvhu river catchment area and the rest of South Africa is severely lacking, we made use of the conventional FAO yield coefficient approach (FAO-56) (Allen et al., 1998) to simulate maize yield as a function of direct precipitation.

$$Y = Y_p[1 - k_y(1 - \frac{ETA}{ETC})]. \quad (22)$$

[5] Maize crop growth is halted at temperatures below 10 degrees Celsius. When the temperature rises above 30 degrees Celsius, maize roots struggle to absorb enough water to keep the crop growing at full pace (Duncan et al., 2010). Equations (20) and (21) are simply conditions to ensure that T_{max} and T_{min} are within the appropriate temperature range for maize crop growth.

[6] Vn represents the nth leaf stage.

The parameters Y, Y_p, k_y, ETA, and ETC represent the simulated crop yield, the crop yield potential when water is not a constraint, the empirical FAO crop yield response factor, the actual crop evapotranspiration, and the crop evapotranspiration when water is not a constraint, respectively. ETA represents the crop evapotranspiration under non-standard conditions (when water is a constraint). For maize, the FAO seasonal yield response factor is 1.25 (Doorenbos and Kassam, 1998). The area's maximum crop yield is assumed to be 6 tons per hectare. This is due to the fact that the average rain-fed maize yield in the area is between 2 and 6 tons per hectare (Lacambra et al., 2020). Environmental factors such as radiation, temperature, humidity, and wind speed, as well as crop type and growth stage, all influence ETC. The FAO single crop coefficient approach ($ETC = k_c \cdot ET_o$) is used to calculate ETC, where k_c and ET_o represent the crop coefficient, and the reference crop evapotranspiration, respectively. ET_o is the evapotranspiration rate from a non-water-stressed reference surface (usually a grass reference crop). The FAO-56 (Allen et al., 1998) method of estimating missing weather data was used to estimate missing meteorological data that are compulsory to determine the yield function. The common complimentary relationship $ETA + ETC = 2 \cdot ETW$ is used to determine ETA (Bouchet, 1963). ETW represents the wet environment evapotranspiration (evapotranspiration from a moisturised surface). The Priestley-Taylor equation is used to calculate ETW (Priestley and Taylor 1972).

$$ETW = \varepsilon \cdot \frac{\eta}{\eta+\gamma}(R_n - G). \tag{23}$$

The parameters ε, η, γ, R_n, and G represent the Priestley-Taylor evaporation coefficient, the slope of the saturation vapor pressure curve at the air temperature, the psychometric constant, the net radiation, and the soil heat flux, respectively. Weather variables such as mean daily temperature, maximum daily temperature, minimum daily temperature, and actual vapor pressure were used to calculate net radiation. Because the amount of the daily or 10-day soil heat flux is very small in comparison to net radiation, it is frequently ignored, resulting in $G \approx 0$ (Allen et al., 1998).

The market output maize price (R2345.07 per ton)[7] was obtained from the South African Grain Information Service (SAGIS). This was the market output maize price for the 2019/2020 season, which is the last season in our empirical analysis. The lowest market output maize price from 2010 to 2020, according to the South African Grain Information Service, is R1,131.50 per ton. To capture low market output price events, we use this value in the sensitivity analysis. For a sample of South African farmers, the risk aversion coefficient was estimated to be 0.169 (Gwata, 2010). Since $r > 0$ and is never equal to 1, we assume that the risk aversion coefficient is equal to 0.05 for the sensitivity analysis to capture low levels of risk aversion. The loading factor for health insurance and access in South Africa ranges from 0.67 to 0.85 (Urban and Streak, 2013). We assume that the loading factor is 0.67 for simplicity and without a loss of generality. We use the value 0.85 for the sensitivity analysis.

In previous studies, the average level of direct payments made to farmers was employed as a proxy for measuring the farmer's initial wealth (Dalhaus and Finger, 2016). Due to a lack of direct payments data for South African maize farmers, we use the farmer’s average level of net income as a proxy to estimate the farmer's initial wealth. An average net income of R26,600.00 per year is reported for a sample of South African small-holder farmers (Zantsi et al., 2019). The average cultivated land area is roughly 2.7 hectares (Zantsi et al., 2019). As a result, we assume that the farmer's initial wealth is R9,851.85. The simplified method of moments was used to estimate the parameters for the Weibull distribution (Faraji et al., 2020). The shape and scale parameters for seasonal station data are 2.45 and 1130.04, respectively. Station data stages 1, 2, 3, and 4 have shape parameters of 2.21, 2.44, 1.04, and 1.33, respectively. Similarly, the scale parameters for stages 1, 2, 3, and 4 of station data are 159.33, 256.9, 228.7, and 443.71, respectively. The shape and scale parameters for seasonal grid data are 3.46 and 601.17, respectively. Grid data stages 1, 2, 3, and 4 have shape parameters of 2.06, 2.24, 1.25, and 1.96, respectively. Similarly, the scale parameters for stages 1, 2, 3, and 4 of grid data are 99.96, 156.36, 119.59, and 228.06, respectively.

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[7] This was the market output maize price for the year 2020, the equivalent in US dollars was 142.38.

Table 1: Economic values of the Luvuvhu catchment area maize production in the base case.

Parameter	Description	Units	Value
p	Price per output	Rand per Ton	2,345.07
r	Risk aversion coefficient	-	0.169
α	Loading factor	Percentage	1.67
θ	Subsidy	Percentage	0
τ	Quantile	Percentage	0.3
m	$f(X)$ shape parameter	-	2.45 (station data) 3.46 (grid data)
ψ	$f(X)$ scale parameter	-	1130.04 (station data) 601.17 (grid data)
w	Initial wealth	Rand per hectare	9,851.85

5. Results and discussions

The section on results is broken into two sub-sections. The first section examines the entire growing season, while the second examines the crop growth stages. Weather station data and grid data will be analyzed, and the efficiency of each outcome (payment and premium) will be compared.

5.1 Entire season

Our results (Table 2) show that, when grid data is used, we have a higher coefficient of determination (represented by R^2 on Table 2), smaller standard error of the coefficient estimates, and a smaller standard error of regression than situations when weather station data is used to promote RIBI contracts. This confirms the assumption made earlier: the use of grid data during the design of the RIBI contract leads to a lower basic risk. Previous studies have found similar conclusions (Dalhaus and Finger, 2016; Bucheli et al., 2020). As a result, index insurance contracts designed by using grid data are more appealing to farmers than contracts designed by using weather station data. This is because neither the insured nor the insurer wants to be associated with a higher basis risk. The insurance company will benefit from the use of grid data in constructing index insurance contracts since the uptake of insurance contracts by farmers will be high. In addition, higher basis risk might deter the reputation of the insurance company and create a situation whereby such incidents can negatively impact some other derivative insurance products sold on the market (life insurance, etc.).

Table 2: Main estimation results of the yield function.

Parameters	Weather station data	Grid data
c (intercept)	0.873799 (0.0006)	0.684796 (0.0267)
Marginal product of water (β)	0.000282 (0.1979)	0.001056 (0.0439)
R^2	0.085631	0.125551
S.E. of regression	0.456656	0.398387
S.E. of coefficient	0.00214	0.000501
Crop water requirement (x_g)	1805.39	661.1

Notes: Parentheses indicate the P-value at a 5% significant level.

Relative to the premium, we found (Table 3) that the optimal premium for seasonal when grid data are used (R600.67) is lower than the optimal premium when weather station data are used (R896.61). This means that in terms of farmers' affordability to purchase rainfall index insurance contracts in the study area, the use of grid data in designing the contract is preferable to the use of weather station data. Furthermore, we ran a sensitivity analysis to simulate the extent to which changes in the selected parameters (the market output price, the insurance loading factor, the crop water requirement, and the marginal product of water) would affect the optimal premium patterns. We observe (Table 3) that the higher (base case values doubled) the crop water requirement (CWR)[8] and the marginal product of water (MPW)[9], the higher the optimal premium to the farmer. When the crop water requirement is increased, there is a greater chance (in drought-prone areas) that there will not be enough rainfall to meet the crop water requirement level, as opposed to when the crop water requirement is low. As a result, raising the crop water requirement increases the insurer's risk, and hence the premium.

Table 3: Main results of the optimal premium function in the base case.

	Premium (Rands)	
	Weather station data	Grid data
Base case	896.61	600.67
Reduction in market output price (p)	432.62	289.83
Increase in the loading factor (α)	993.25	665.42
Increase in the Crop Water Requirement (x_g)	2880.9	3232.5
Increase in the Marginal Product of Water (β)	1793.2	1201.3

In addition, we increase the loading factor (Table 3) to investigate the differences in the optimal premium for insurers with higher internal expenses, taxes, and profit margins versus insurers with lower expenses, taxes, and profit margins (base case scenario). We found that the higher the insurance loading factor, the more expensive is the premium. Relative to the market output price, we reduce it to account for non-drought events in which the market price is lower because the supply of outputs exceeds the demand. The lower the market output price, the lower the optimal premium (Table 3). This means that index insurance contracts are more expensive to purchase in drought-prone areas, where supply is often likely to be far less than demand.

[8] This is the threshold value x_g that triggers the insurance payment to the farmer if and only if the cumulative rainfall index (X) falls below this value x_g. If the cumulative rainfall index (X) is greater than or equal to x_g, the farmer receives no compensation, regardless of whether or not the farmer has suffered losses.

[9] This is the crop-water yield function's slope coefficient β. Because it represents the change in yield per 1 mm change in water (rainfall X), it is referred to as the marginal product of water.

The insurance payment that maximizes the farmer's expected utility of final wealth is defined as the optimal payment to the farmer. The option with the highest optimal payment pattern (use of grid data or the use of weather station data in the contract's design) is defined as the most preferred option to farmers. Drought is represented by lower rainfall index values (Figure 3), as high values indicate a large amount of rainfall received during that rainfall period. The closer the rainfall index gets to zero, the worse the drought becomes. As a result, we found (Figure 3) that when the drought is severe, the use of grid data when designing the contract is the farmer's most preferred option in terms of optimal payment. Only during low-intensity drought events may farmers prefer contracts designed using weather station data. In addition, we found that when grid data are used farmers tend to receive a higher payment (Figure 3) at a lower premium (base case in Table 3) than situations when weather station data are used.

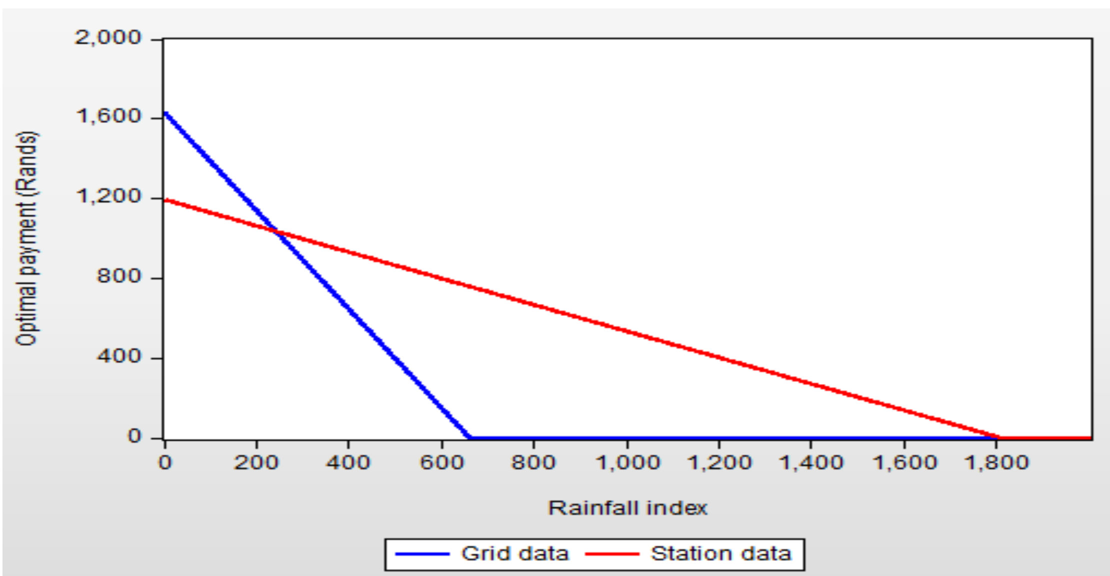

Figure 3: The optimal payment function for seasonal weather station and grid data in the base case.

Furthermore, we ran a sensitivity analysis to simulate the extent to which changes in the selected parameters (the market output price, the crop water requirement, and the marginal product of water) would affect the optimal payment patterns. We found (Figure 4) that the higher the crop water requirement and the marginal product of water, the higher the optimal payment to the farmer. Relative to the marginal product of water (MPW), for instance, we found that when grid data are used farmers tend to receive a higher payment (Figure 4) at a lower premium (Table 3) than when weather station data is used to confirm the drought damage. Relative to the crop water requirement (CWR), the use of grid data is associated with a higher payment to the farmer at a higher premium than situations when weather station data are used. The lower the market output price, the smaller the payment to the farmer (Figure 4). The theoretical analysis has already validated these findings.

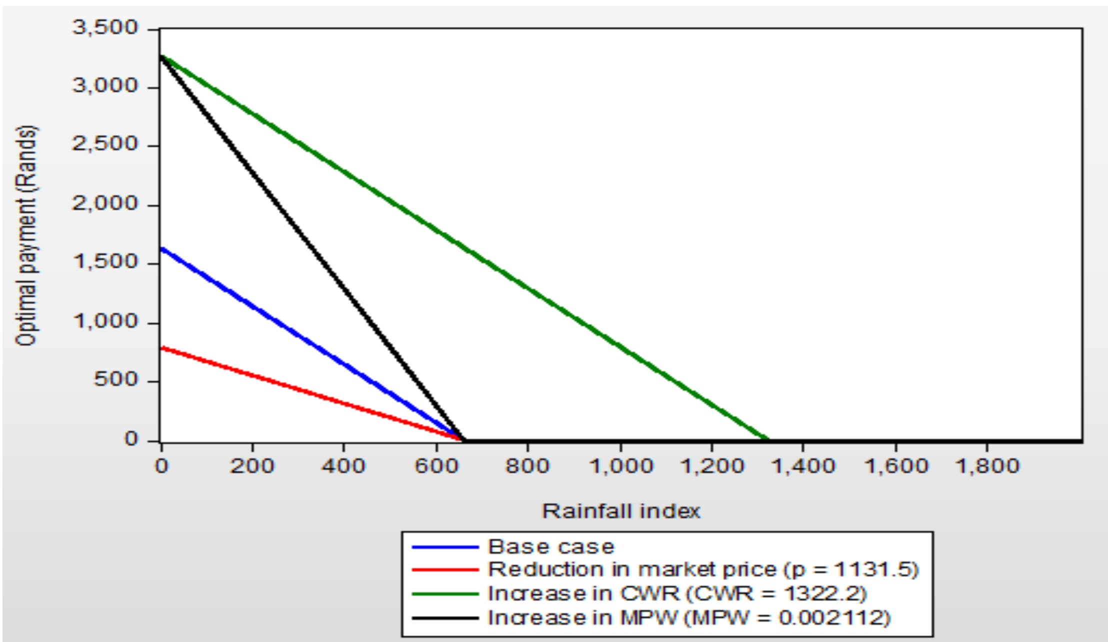

Figure 4: The optimal payoff function for seasonal grid data in the sensitivity case.

Because all of the parameters (CWR, MPW, market output price) involved in computing the optimal payment are positive for both weather station and grid data, changing one parameter when using station data has the same effect on the insurance payment as changing the same parameter when using grid data. This has been tested for both grid and station datasets. As a result, the sensitivity analysis of the parameters will only be shown using grid data. The rationale for this is that grid data is less affected by basis risk, and thus, provides more accurate maize yield projections in the area.

Table 4: Main results of the necessary condition for the farmer to purchase the insurance policy.

	Expected utility	
	Weather station data	Grid data
Base case	-3138.9	-2472.8
Reduction in market output price (p)	-2760.8	-2167.5
Increase in the loading factor (α)	-3139.2	-2476
Increase in the Crop Water Requirement (x_g)	-3186.9	-3302.1
Increase in the Marginal Product of Water (β)	-3262.1	-2673.4
Increase in the subsidy constant (θ)	-3138.6	-2469.4
Reduction in the risk aversion coefficient (r)	-8523.6	-6722.7

The difference in the expected utility (Table 4) is the condition that a farmer must meet to consider acquiring an index insurance contract (inequality in Equation (12)). This is a participation constraint in a decision theory framework. This is expressed as the expected utility when a farmer purchases an insurance policy minus the expected utility when she does not (the difference between the two expected utilities). The use of both grid and weather station seasonal data shows that acquiring insurance protection provides the farmer with a lower expected utility than not having one because the difference in Equation (12) is less than zero in both cases. When comparing seasonal weather station data to seasonal grid data, we found that acquiring an insurance policy created using seasonal grid data always has the biggest negative difference, hence generating more losses. This indicates that the expected utility of obtaining an insurance policy based on grid data is always higher than that of purchasing an insurance policy based on weather station data, even though being uninsured still leads to a higher expected utility than being insured.

Furthermore, we ran a sensitivity analysis to simulate the extent to which changes in the selected parameters (the market output price, the crop water requirement, loading factor, subsidy constant, risk aversion coefficient, and the marginal product of water) would affect the expected utility for the farmer when purchasing an insurance policy. When seasonal weather station and grid data are used in contract design, the expected utility criterion (in Table 4) increases (having lower negative values than the base cases) when the market output price drops and the subsidy constant increases. This means that, relative to the subsidy constant, the larger the subsidy, the higher the farmer's expected utility from acquiring insurance protection. When the loading factor is large, the expected utility criterion is expected to be higher (having lower negative values than the base cases) since the farmer is charged more, i.e. a higher premium. We also reduced the risk aversion coefficient to accommodate slightly risk-averse farmers. We found that the expected utility decreases (having higher negative values than the base case). This means that less risk-averse farmers will obtain the least expected utility from purchasing insurance protection.

5.2 Crop growth stages.

Our results (Table 5) show that, when grid data is used, we have a higher coefficient of determination (represented by R^2 on Table 5), and a smaller standard error of regression per growth stage than situations when weather station data is used to promote RIBI contracts. Only in stage 4 does the use of grid data result in a lower coefficient of determination than the use of weather station data. However, we found (Table 5) that using grid data per growth stage minimizes basis risk considerably better than using weather station data for the other three stages.

Table 5: Main estimation results of the yield function per growth stage.

Parameters	Weather station data			
	Stage 1	Stage 2	Stage 3	Stage 4
c (intercept)	0.959689 (0.001)	1.118944 (0.0002)	0.948363 (0.0000)	0.952569 (0.0000)
Marginal product of water (β)	0.001372 (0.3547)	0.000150 (0.9012)	0.000764 (0.2988)	0.000465 (0.1053)
R^2	0.028045	0.002400	0.025468	0.125769
S.E. of regression	0.480335	0.481791	0.488904	0.481007

S.E. of coefficient	0.001459	0.001194	0.000722	0.000278
CWR (x_g)	308.4759	1759.83	568.79	925.4817
Parameters	Grid data			
	Stage 1	Stage 2	Stage 3	Stage 4
c (intercept)	0.895279 (0.0002)	0.926876 (0.0010)	0.949893 (0.0000)	0.991899 (0.0005)
Marginal product of water (β)	0.003782 (0.0826)	0.001614 (0.2858)	0.001591 (0.3414)	0.001023 (0.3587)
R^2	0.139761	0.103265	0.062094	0.040975
S.E. of regression	0.406784	0.472490	0.475441	0.468527
S.E. of coefficient	0.002104	0.001484	0.001645	0.001097
CWR (x_g)	128.94	282.55	272.17	382.23

Notes: Parentheses indicate the P-value at a 5% significant level.

Relative to the premium, we found (Figure 5) that when crops are insured according to growth stages (using grid data and weather station data), the total optimal premium for the entire season is very expensive compared to when growth stages are not taken into account (Table 3). Furthermore, we found (Figure 5) that, when compared to using weather station data, the use of grid data provides the cheapest premium in most growth stages (except for stage 2). This means that in terms of farmers' affordability to purchase rainfall index insurance contracts per growth stage, the use of grid data in designing the contract is preferable to the use of weather station data. We will use the findings of using grid data to make conclusions on the optimal premium in the area since using grid data effectively reduces basis risk.

Relative to the premium when grid data is used, stage 3 (flowering) is the most expensive, followed by stage 2 (vegetative), stage 4 (yield formation and ripening), and finally stage 1 (establishment). Stage 1 should be less expensive (relative to the farmer) because there is a lower impact associated with drought, much effort has not been spent yet on crop production in this stage compared to the final stages. However, because a greater portion of the growth cycle is yet to be observed and no information on how the yield might turn out, there is a high risk associated with Stage 1. As a result, stage 1 should be quite costly (relative to the insurer) to compensate the insurer for the huge risk being transferred to her. Farmers gain from this kind of premium pricing. The insurer gains from stage 3 (most expensive), which is associated with reduced risk and a larger impact, while the insured is paid for the high impact she is exposed to.

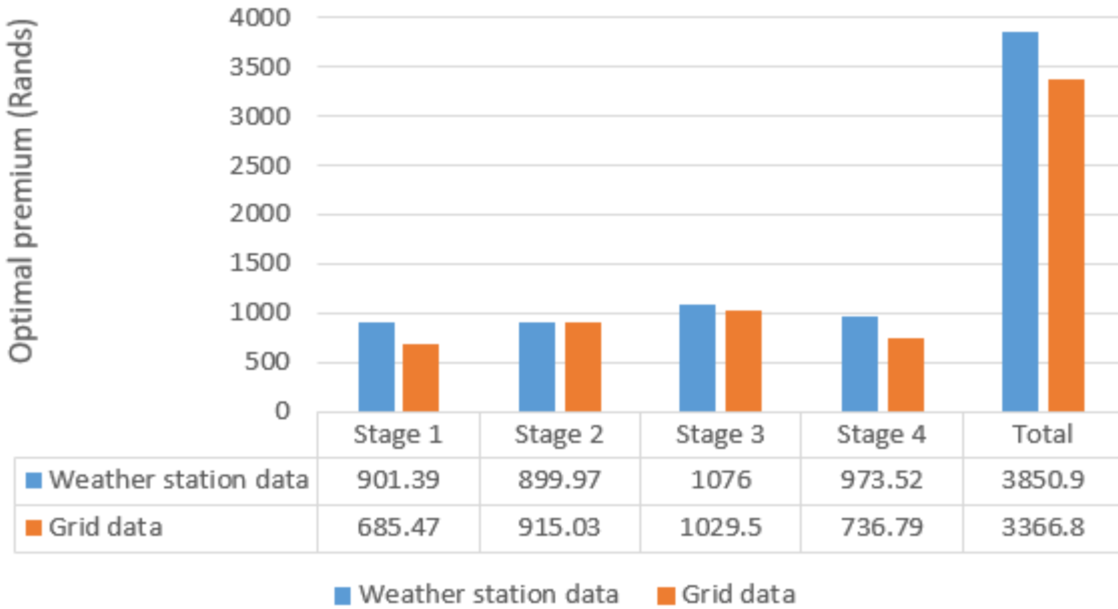

Figure 5: The optimal premium when weather station and grid data are used per crop growth stage (in the base case).

It's worth noting that the same optimal premium function that is utilized for seasonal data is also used for growth stage data (See equation (11) and (17)). Because all the parameters (CWR, MPW, market output price) involved in computing the optimal payment are positive for both weather station and grid data, changing one parameter has the same effect on the insurance premium as shown for the seasonal premium function. This holds for both the optimal payment and the expected utility criterion. As a result, no sensitivity analysis of the parameters will be shown in this section.

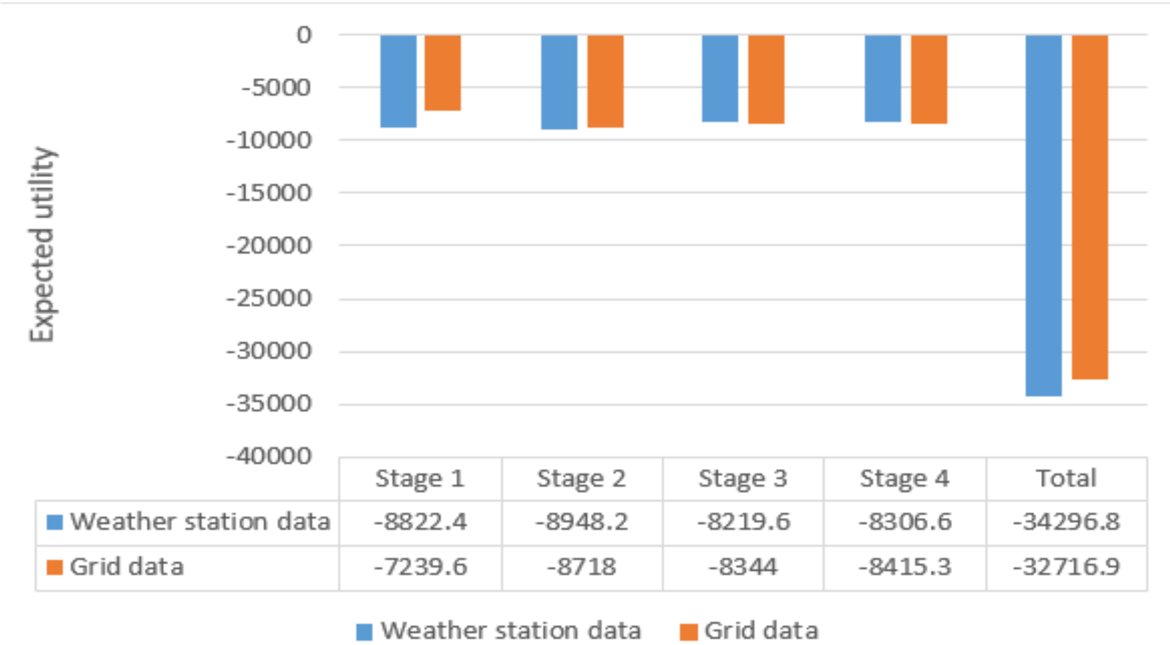

Figure 6: The necessary condition for the farmer to purchase the insurance policy per crop growth stage (in the base case).

When growth stages are taken into account, the use of both grid and weather station seasonal data shows that acquiring insurance protection provides the farmer with a lower expected utility than not having one because the difference in Equation (18) is less than zero in both cases (Figure 6). Furthermore, when growth stages are taken into account (using grid data and weather station data), the total expected utility (Figure 6) is very lower (having higher negative values) compared to when growth stages are not taken into account (Table 4). This indicates that the total expected utility of obtaining an insurance policy based on growth stages is always higher than that of purchasing an insurance policy based on seasonal data, even though being uninsured still leads to a higher expected utility than being insured. In the same vein, grid data were used to analyze specific growth stages since they minimize basis risk more effectively. Stage 1 has the highest difference (having the lowest negative value), followed by stage 3, stage 4, and stage 2, which has the lowest difference. This demonstrates that stage 1 provides farmers with the highest expected utility from purchasing insurance protection, followed by stage 3, stage 4, and stage 2.

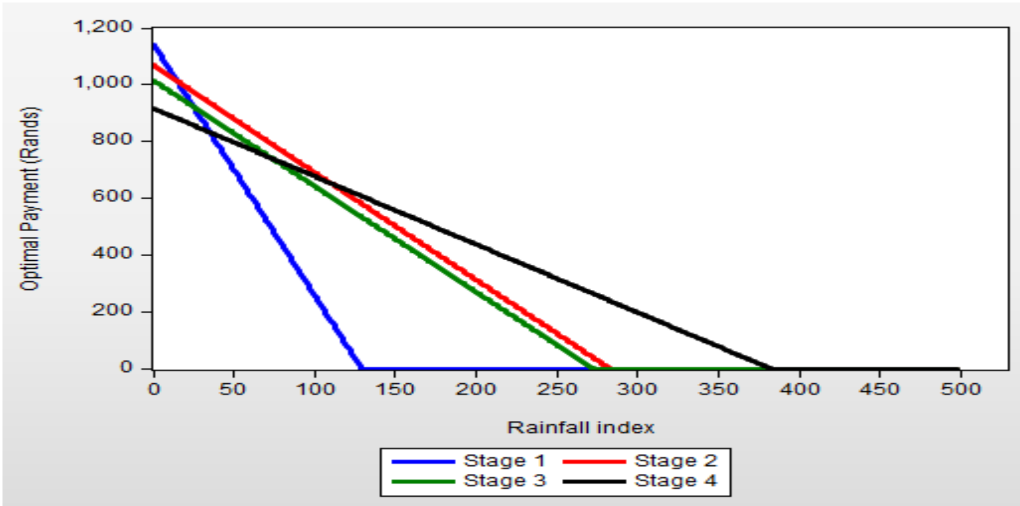

Figure 7: The optimal payoff function per growth stage when grid data are used.

We can recall (from Table 5) that stage 1 has the highest marginal product of water (0.003782 ton per hectare per mm) when grid data are used, followed by stage 2 (0.001614 ton per hectare per mm), stage 3 (0.001591 ton per hectare per mm), and finally stage 4 (0.001023 ton per hectare per mm). As a result, we found (Figure 7) that stage 1 provides the smallest compensation to the farmer, followed by stages 3 and 2, except in the worst-case drought events. That is, as the drought worsens (as the index approaches zero), the stage with the highest marginal product of water provides a higher compensation to the farmer. Similarly, the farmer receives the lowest payment during the growing stage with the lowest crop water requirement. From Table 5, stage 1 requires the least amount of crop water (128.94 mm), followed by stages 3 (272.17 mm), 2 (282.55 mm), and 4 (382.23 mm). This is justifiable because if the growth stage requires a small amount of water, it is less sensitive to water stress. As a result, the farmer's payment at that stage of crop growth is likely to be small.

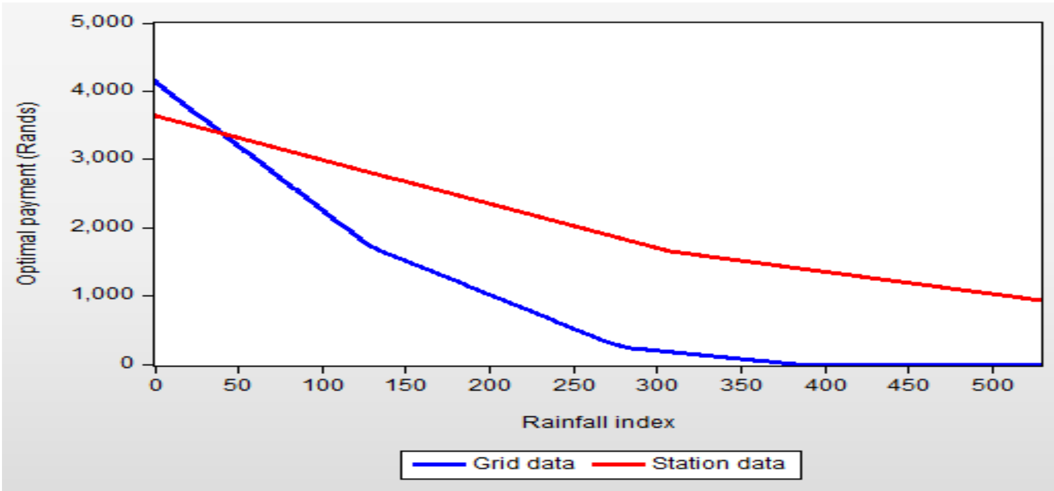

Figure 8: The total optimal payment (using weather station and grid data) when growing stages during the season are considered (in the base cases).

As mentioned previously, drought is represented by lower rain index values, as high values indicate a large amount of rainfall received during that growing season. The closer the rain index gets to zero, the worse the drought becomes. Grid data still generates a larger overall payment to farmers for worst-case drought events when a farmer insures her crops according to growing stages (Figure 8). Only during drought events of low intensity can weather station rainfall data result in a larger overall optimal payment to farmers according to Figure 8.

6. Conclusion and Policy Implications

The paper examined the financial viability of RIBI in terms of the payment of a risk-mitigating strategy against drought. With an empirical application to the Luvuvhu river catchment, we investigated a rainfall index insurance contract. We have shown that optimal insurance payment to the farmer does exist. We further found that the use of grid data minimizes basis risk more effectively than the use of station data. In addition, we discovered that the most advantageous rain index insurance policy for maize farmers in this area is one in which crops are insured according to growth stages and the index is calculated using grid data. These findings have significant policy implications for crop insurance policymakers in South Africa. The Luvuvhu catchment area, for example, is not densely populated with rain gauges. As a result, policymakers may want to consider requiring insurance companies to use grid data for index insurance products to be beneficial to farmers. Grid data also provides complete datasets for weather variables without inconsistencies. Grid and phenology data needed for estimating drought risks for individual farms should be freely available, as this is one of South Africa's key problems. South African institutions must empower agricultural extension officers to ensure that this type of information is shared with the rest of the country. This is expected to raise the demand for RIBI and aid in mitigating the effects of the drought on farmers and society at large.

Furthermore, crop insurance coverage is still acquired throughout the entire growing season in South Africa. Based on these findings, policymakers may want to consider enacting rules that require insurance companies to deploy crop insurance policies that take into account individual crop growth stages. As we have shown from the findings, insuring maize according to individual growth stages proved to be more effective than insuring it over the full growing season. Policymakers might also want to raise awareness of crop insurance products characterized by crop growth stages, as most farmers and insurance companies in the country are lagging in terms of these types of insurance products. The technique of calculating a premium is also important in determining the effectiveness of an insurance contract. We used the expected value of the insurance payment approach to price our premium. We found that the insurer gains from premium pricing in some growth stages (stage 3), i.e., receiving a higher premium from farmers when there is lower risk associated with that stage. In terms of the insured, she also gains from premium pricing in certain growth stages (stage 1), such as paying a lower premium to the insurer when that stage is associated with a higher risk. Different results may be obtained if a different pricing approach is used.

South African crop insurance policymakers should explore enacting rules that will allow the government and private sector to subsidize crop insurance premiums, possibly based on the level of risk associated with the growth stage covered by the insurance policy. This is especially important since crop insurance is currently not subsidized at all. The lack of a subsidy is one aspect cited as contributing to the country's reduced demand for crop insurance products. Our empirical findings show that a bigger subsidy boosts the expected utility of farmers in the Luvuvhu catchment area. However, we found that purchasing an insurance plan provides the farmer with a lower expected utility than not purchasing one. This is not to say that farmers should not buy insurance, they can suffer drought damage and receive compensation from the insurer, and the insurance contract has lower premiums than standard crop insurance contracts. Finally, we found that the total expected utility of obtaining an insurance policy based on growth stages is always higher than that of purchasing an insurance policy based on seasonal data. In conclusion, we argue that a rainfall index-based insurance contract can be a viable risk management tool for hedging drought risks in the Luvuvhu catchment area.

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Appendix

Proof of Theorem (1.)

Let $s(X)$ be the state variable which describes the state of the farmer's terminal wealth when an outcome value X is realised, be defined by

$$s(X) = \int_0^X \alpha h(u) f(u) du. \tag{24}$$

Suppose that $g(X,u) = \alpha h(u) f(u)$ and $g_X(X,u)$ are continuous for $u \in [0,X]$. Then we differentiate $s(X)$ by applying the Leibniz integral rule as follows

$$\dot{s}(X) = \frac{d}{dX}\left[\int_0^X \alpha h(u) f(u) du\right] = \int_0^X \frac{\partial}{\partial X} \alpha h(u) f(u) du = \alpha h(X) f(X).$$

To obtain the boundary conditions, we use Equation (24) to get the following

$$s(0) = \int_0^0 \alpha h(u) f(u) du = 0,$$

$$s(\bar{x}) = \int_0^{\bar{x}} \alpha h(u) f(u) du = z \text{ byEquation}(7).$$

Proof of Proposition (1.)

The hamiltonian function of the system (6), (8), (9) is given as follows

$$\mathcal{H}(s,h,\lambda,\mu) = U[w + py(X) + h(X) - (1-\theta)z] f(X) + \lambda \alpha h(X) f(X) + \mu h(X), \tag{25}$$

Hence, the first order necessary conditions are as follows

$$\mathcal{H}_h(s,h,\lambda,\mu) = U'[w + py(X) + h(X) - (1-\theta)z] f(X) + \lambda \alpha f(X) + \mu = 0. \tag{26}$$

$$\mathcal{H}_\mu(s,h,\lambda,\mu) = h(X) \geq 0, \mu \geq 0, \mu h(X) = 0. \tag{27}$$

$$\dot{\lambda} = 0. \tag{28}$$

$$\dot{s} = \alpha h(X) f(X),\ s(0) = 0,\ s(\bar{x}) = z. \quad (29)$$

From Equation (26), we obtain the value for $h(X)$ as follows.

$$h(X) = U'^{-1}[-\frac{\lambda \alpha f(X) + \mu}{f(X)}] + (1-\theta)z - py(X) - w. \quad (30)$$

From Equation (28), we determine the value for the costate variable as follows

$$\int \frac{d\lambda(X)}{dX}\, dX = \int 0\, dX \Longrightarrow \lambda(X) = c. \quad (31)$$

Where c is a constant of integration. Therefore, $\lambda(X)$ is a constant. The costate variable represents the farmer's shadow cost of acquiring the insurance contract. The higher the protection being offered by the contract, the higher the premium. Hence, the shadow cost is the cost to the farmer for an additional unit of insurance protection. Intuitively, the payment $h(X)$ should be equal to zero for all X values in the set $[x_g, \bar{x}]$ contained in the support of X. The latter implies that the farmer receives no payment since the observed total rainfall throughout the contract was higher or equal to the threshold. Likewise, for all $X \in [0, x_g)$, the payment should be strictly positive. The latter implies that the farmer receives a payment since the observed index value is strictly less than the agreed threshold. Thus, there was a low rainfall compared to the threshold. To sum up from the aforementioned observations, a reasonable structure of the optimal payment function should be $h(X) = 0\ \forall\ X \in [x_g, \bar{x}]$, and $h(X) > 0\ \forall\ X \in [0, x_g)$. As a result, for the case when $X \in\ [x_g, \bar{x}]$, we have that

$$h(X) = 0 \Longrightarrow U'^{-1}[-\frac{\lambda \alpha f(X) + \mu}{f(X)}] + (1-\theta)z - py(X) - w = 0. \quad (32)$$

Since $\lambda = c$, we then solve for μ from Equation (32) to get

$$\mu(X) = -U'[w + py(X) - (1-\theta)z] f(X) - c\alpha f(X),\ \forall\, X\ \in [x_g, \bar{x}]. \quad (33)$$

To find the value for c, recall the second case when $h(X) > 0\ \forall\ X \in [0, x_g)$. The latter implies that $\mu(X) = 0\ \forall\ X \in [0, x_g)$ by Equation (27). From the aforementioned hypothesis, the optimal payment function is continuous for all X in the set $[0, \bar{x}]$. Hence, $\mu(X)$ is thus also continuous for all X values (Caputo, 2005, p.105). Consequently, $\mu(X)$ is continuous at the switching value $X = x_g$ and thus we have that

$$\mu(x_g) = -U'[w + py(x_g) - (1-\theta)z]f(x_g) - c\alpha f(x_g) = 0. \tag{34}$$

Solving for c from the above equation we get the following

$$c = -\frac{U'[w+py(x_g)-(1-\theta)z]}{\alpha}. \tag{35}$$

Hence,

$$\lambda(X) = -\frac{U'[w+py(x_g)-(1-\theta)z]}{\alpha} \ \ \forall\, X \in [0, \bar{x}], \tag{36}$$

and

$$\mu(X) = \begin{cases} 0 \qquad \forall X \in [0, x_g), \\ f(X)[U'[w + py(x_g) - (1-\theta)z] - U'[w + py(X) \\ -(1-\theta)z]] \qquad \forall X \in [x_g, \bar{x}]. \end{cases} \tag{37}$$

Consequently, it should be noted that on the interval $[x_g, \bar{x}]$, $\mu(X)$ is only equal to zero at an instant, that is at $X = x_g$. Therefore, substituting the value for $\mu(X)$, and that of $\lambda(X)$ from Equation (36) into Equation (30) yields the optimal solution in Equation (10). Furthermore, from Equation (10), it is clear that $h(X) = 0 \ \forall X \in [x_g, \bar{x}]$, and $h(X)$ is continuous for all X as asserted.

Proof of Proposition (2.)

$$z^* = \alpha \int_0^{\bar{x}} h^*(X) f(X) dX \tag{38}$$

$$= \alpha[\int_0^{x_g} p\beta(x_g - X) f(X) dX + \int_{x_g}^{\bar{x}} 0 \cdot f(X) dX] \tag{39}$$

$$= p\alpha\beta[\int_0^{x_g} x_g f(X) dX - \int_0^{x_g} X f(X) dX] \tag{40}$$

$$= p\alpha\beta[x_g[F(x_g) - F(0)] - \mathbb{E}[X_{[0,x_g)}]] \tag{41}$$

$$= p\alpha\beta[x_g F(x_g) - \mathbb{E}[X_{[0,x_g)}]]. \tag{42}$$

We know that $\int_{x_g}^{\bar{x}} 0 \cdot f(X) dX = 0 \cdot \int_{x_g}^{\bar{x}} f(X) dX = 0$, and $F(0) = 0$. Hence, proved.

Proof of Proposition (3.)

The hamiltonian function of the system (13), (14), (15) is given as follows

$$\mathcal{H}(s,h_i,\lambda_i,\mu_i) = U[w + p\sum_i y(X_i) + \sum_i h(X_i) - (1-\theta)z]f(X_i)$$
$$+\lambda_i\alpha\sum_i h(X_i)f(X_i) + \mu_i\sum_i h(X_i). \quad (43)$$

Hence, the first order necessary conditions are as follows

$$\mathcal{H}_h(s,h_i,\lambda_i,\mu_i) = U[w + p\sum_i y(X_i) + \sum_i h(X_i) - (1-\theta)z]mf(X_i)$$
$$+\lambda_i m\alpha f(X_i) + m\mu_i = 0. \quad (44)$$

$$\mathcal{H}_{\mu_i}(s,h_i,\lambda_i,\mu_i) = \sum_i h(X_i) \geq 0, \mu_i \geq 0, \mu_i\sum_i h(X_i) = 0. \quad (45)$$

$$\dot{\lambda} = 0. \quad (46)$$

From Equation (44), we obtain the value for $\sum_i h(X_i)$ as follows.

$$\sum_i h(X_i) = U'^{-1}\left[-\frac{\lambda_i\alpha mf(X_i)+m\mu_i}{mf(X_i)}\right] + (1-\theta)z - p\sum_i y(X_i) - w. \quad (47)$$

From Equation (46), we determine the value for the costate variable as follows

$$\int \frac{d\lambda(X_i)}{dX_i}\,dX_i = \int 0\,dX_i \Longrightarrow \lambda(X_i) = c_i. \quad (48)$$

Where c_i is a constant of integration. Therefore, $\lambda(X_i)$ is a constant. The costate variable represents the farmer's shadow cost of acquiring the insurance contract. The higher the protection being offered by the contract, the higher the premium. Hence, the shadow cost is the cost to the farmer for an additional unit of insurance protection. Intuitively, the payment $\sum_i h(X_i)$ should be equal to zero when all the X_i values are in the set $[x_{gi}, \bar{x}_i]$ contained in the support of each index X_i. The latter implies that the farmer receives no payment since the observed total rainfall in each growth stage was higher or equal to the respective threshold x_{gi}. Likewise, for any (for at least one index of the indices) $X_i \in [0, x_{gi})$, the payment $\sum_i h(X_i)$ should be strictly positive. The latter implies that the farmer receives a payment since one of the observed growth stages' indices is strictly less than that growth stage's threshold. Thus, there was a low rainfall in that growth stage compared to its threshold. To sum up from the aforementioned observations, a reasonable structure of the optimal payment function should be $\sum_i h(X_i) = 0 \ \forall \ X_i \in [x_{gi}, \bar{x}_i]$, and $\sum_i h(X_i) > 0 \ for\ any\ X_i \in [0, x_{gi})$. As a result, for all $X_i \in [x_{gi}, \bar{x}_i]$, we have that

$$\sum_i h(X_i) = 0$$

$$\Rightarrow U'^{-1}[-\frac{\lambda_i \alpha m f(X_i) + m\mu_i}{m f(X_i)}] + (1-\theta)z - p\sum_i y(X_i) - w = 0. \tag{49}$$

Since $\lambda_i = c_i$, we then solve for μ_i from Equation (49) to get

$$\mu(X_i) = -U'[w + p\sum_i y(X_i) - (1-\theta)z]f(X_i) - c_i\alpha f(X_i),\ \forall X_i \in [x_{gi}, \bar{x}_i]. \tag{50}$$

To find the value for c_i, recall the second case when $\sum_i h(X_i) > 0$ for any (for at leat one index of the indices) $X_i \in [0, x_{gi})$. The latter implies that $\mu(X_i) = 0 \ for\ any\ X_i \in [0, x_{gi})$ by Equation (45). From the aforementioned hypothesis, the optimal payment function is continuous for all X_i in the set $[0, \bar{x}_i]$. Hence, $\mu(X_i)$ is thus also continuous for all X_i values (Caputo, 2005, p.105). Consequently, $\mu(X_i)$ is continuous at the switching value $X_i = x_{gi}$ and thus we have that

$$\mu(x_{gi}) = -U'[w + p\sum_i y(X_{gi}) - (1-\theta)z]f(x_{gi}) - c_i\alpha f(x_{gi}) = 0. \tag{51}$$

Solving for c_i from the above equation we get the following

$$c_i = -\frac{U'[w + p\sum_i y(X_{gi}) - (1-\theta)z]}{\alpha}. \tag{52}$$

Hence,

$$\lambda(X_i) = -\frac{U'[w + p\sum_i y(X_{gi}) - (1-\theta)z]}{\alpha} \ \ \forall X_i \in [0, \bar{x}_i], \tag{53}$$

and

$$\mu(X_i) = \begin{cases} 0 & \forall X_i \in [0, x_{gi}), \\ f(X_i)[U'[w + p\sum_i y(X_{gi}) - (1-\theta)z] & \\ -U'[w + p\sum_i y(X_i) - (1-\theta)z]] & \forall X_i \in [x_{gi}, \bar{x}_i]. \end{cases} \tag{54}$$

Therefore, substituting the value for $\mu(X_i)$, and that of $\lambda(X_i)$ from Equation (53) into Equation (47) yields the optimal solution in Equation (16). Furthermore, from Equation (16), it is clear that $\sum_i h(X_i) = 0 \ \forall X_i \in [x_{gi}, \bar{x}_i]$, and $\sum_i h(X_i)$ is continuous for all X_i as asserted.