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Summary:
Wastewater has become a valuable resource in many regions of the world that face increased level of freshwater scarcity. Reuse of treated wastewater is associated with high economic benefit, but it also leads to pollution of the environment and water bodies. As such, wastewater reuse requires rules of allocation among competing sectors. In this paper, we develop a regional multi-sectoral model of water quantity-quality interaction among the urban, agricultural, and environmental sectors. Our interest lies in the feasibility of reuse, rather than the stability of the regional arrangements, therefore we apply a social planner's approach to the regional problem. Of the various options facing the regional decision-maker, our theoretical analysis demonstrates that the use of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits. An illustrative example provides a numerical solution, using data and results from the literature, to repeat and reinforce the general theoretical relationships.

Acknowledgements: We thank John Burr and Edward Grangetto from the Escondido Growers for Agricultural Preservation (EGAP) for earlier discussions, which led to the development of this study. We would also like to thank Yacov Tsur and Konstantinos Tsagarakis for reviewing and providing comments on an earlier version of this working paper. Ami Reznik wants to express gratitude to the Vaadia-BARD Postdoctoral Fellowship (No. FI-563-2017) for providing supplemental funding for this research. Ariel Dinar acknowledges the financial support by the Hatch Project W3190 "Management of water in a scarce world."

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Abstract

Wastewater has become a valuable resource in many regions of the world that face increased level of freshwater scarcity. Reuse of treated wastewater is associated with high economic benefit, but it also leads to pollution of the environment and water bodies. As such, wastewater reuse requires rules of allocation among competing sectors. In this paper, we develop a regional multi-sectoral model of water quantity-quality interaction among the urban, agricultural, and environmental sectors. Our interest lies in the feasibility of reuse, rather than the stability of the regional arrangements, therefore we apply a social planner's approach to the regional problem. Of the various options facing the regional decision-maker, our theoretical analysis demonstrates that the use of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits. An illustrative example provides a numerical solution, using data and results from the literature, to repeat and reinforce the general theoretical relationships. [155 words]

Key words: wastewater reuse, optimal allocation, social planner, quantity-quality management, externalities

JEL: C61, Q15, Q25, Q53

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1. Introduction

Water resources have become increasingly scarce and their supply more volatile. A quick look at global population dynamics data (United Nations, Department of Economic and Social Affairs, 2006; Demographia World Urban Areas 12th Annual Edition, 2016) reveals that urban population increases over time. To demonstrate, the urban population in 2014 accounted for nearly 55 percent of the total global population, up from one third in 1960. Such increase in urban population added about 2.5 billion people to cities over the past 55 years. These growing urban centers produce sewage that needs to be treated and disposed of at high cost to the society. Alternatively it bears an opportunity cost associated with health concerns, which also result in high cost to society. A common practice in developed countries is that the urban centers follow the state regulations for treatment level, and transport the wastewater for disposal in a river, or the ocean—a costly operation involving energy, infrastructure, and environmental damage. Indeed, treating urban sewage is costly, but discharging it untreated is also costly (Hernández-Sancho et al., 2015).

It has been argued that while wastewater has a great potential (e.g., year-round availability, fertilizer cost-saving) in food, feed, and fish production at different scales, not all countries treat urban sewage, and even fewer countries re-use treated wastewater (Sato et al., 2013). While the global number of wastewater treatment plants and their capacity increased between 1990-1998 and 1999-2013 from 18,062 to 72,007 (FAO, 2016), in many countries and over time, wastewater has remained a source of pollution. It is estimated that 80 percent of all wastewater is being discharged untreated into the world's waterways.

Several alternatives could prove to be attractive for society, such as the use of treated wastewater locally for irrigation. Reznik et al. (2017) show that the adoption of treated wastewater irrigation strategy benefits society through two subsequent routes. It's decreasing the competition over natural freshwater resources, and subsequently delays (or even eliminates) the need for investment in expensive water supply projects (e.g., reservoirs, sea water desalination). However, their analysis, which adopts a central planner approach, ignores environmental consequences, and focuses on the case of Israel—a unique economy in terms of its water institutions and competing sectors for treated wastewater. Other previous work evaluating the economic benefits and costs of wastewater re-use in irrigated agriculture (e.g., Dinar and Yaron, 1986; Dinar et al., 1986; Hussain et al., 2001; Winpenny et al., 2010; Kanyoka and Eshtawl, 2012), focused on maximizing the welfare of the agricultural sector subject to physical and regulatory constraints of wastewater treatment. The models used assumed a given quantity and cost of treatment per volume unit of treated wastewater. Some works included also the environment as a subsector, but with a priori imposed quality standards to be met by society in order to minimize damage. Feinerman et al. (2001) address the issue of who should pay for the disposal of wastewater in their effort to resolve the cost burden allocation between wastewater producers (i.e., the city) and consumers (i.e., farmers). Using a conceptual regional model that also facilitates negotiations, and an illustrative example from a coastline region in Israel, the authors reach the conclusion that the "polluter pays" principle could not be supported. Goldfarb and Kislev (2007), reached a similar conclusion using a steady-state analysis of a sustainable salt regime for the coastal aquifer in Israel.

We depart from both of these studies, and the others cited above, in several major aspects: (1) we endogenize both effluent quantity and quality in our model; (2) we introduce a dependency between the farmers and the city in the form of a shared water source, therefore allowing a greater flexibility in finding a solution, as now tradeoffs between the two types of water (treated wastewater and groundwater) can be accounted for; and (3) our analysis is dynamic and considers environmental quality implications explicitly through the modeling of groundwater aquifer responses to natural changing conditions and economic agents' behavior. By doing so, we are enabling the internalization of externalities for all agents involved.

The main aim of this paper is to address the role of treated wastewater reuse in an economy that is characterized by growing scarcity of natural freshwater resources. We therefore develop a conceptual model to allow us to address most of the omitted issues we mentioned earlier. To simplify, we start with a model that represents a region composed of decision-makers: a city manager and an agricultural grower, and their impact on the environment. The environment could be subject to likely negative impacts on the part of the city or the agricultural sector. In our model, the environment is represented both by a waterway (e.g., dry riverbed or a flowing river) which could be subject to direct disposal of treated wastewater, and a groundwater basin which can be indirectly affected by the use of treated wastewater in agricultural irrigation, due to deep perculation (the term "environment" would be used interchangeably for these two representations throughout the article). In this work, we explicitly define the dynamic equations of motion that control the two important states of the groundwater—water quantity and quality. By doing so, we account in our framework for the negative externalities resulting from changes in the level of water in the aquifer, and in the quality of the wastewater used for irrigation. The latter can have a negative impact on the quality of water in the aquifer, due to deep percolation. We use a social planner's approach to the regional problem of water quantity-quality effects on urban net income and agricultural productivity. Of the various options facing the regional decision-maker, the social planner's approach demonstrates that the use of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits. That 'first-best' solution is an ideal benchmark and, as discussed later in the paper, private competitive solutions would depart from it, leading to economic welfare losses for agents in the region. While there is room for contribution to the debate over the effectiveness of policymaking and different institutions to support the efficient solution, we leave that task to future research.

The paper proceeds as follows: next, we develop the model framework and individual components, and demonstrate how they are linked. We derive several general results to be tested in the section dealing with the illustrative application in Section 3. Section 4 concludes and introduces several regional policy implications of treated wastewater use in irrigated agriculture, including regulations of externalities, promotion of technology adoption, and investment in R&D.

2. Setting the Framework

We start by developing a modeling framework that integrates the demand for and supply of treated wastewater with the physical medium of its application. This, in turn, would allow the calculation of private and social benefits, and costs that are associated with different decisions regarding the production and use of treated wastewater.

2.1 Model

Consider a region that is characterized by a group of agents (water users and decision-makers) and water sources. The region includes an urban sector (a city) that is represented by an aggregated consumer's utility function; it owns a wastewater treatment plant (WWTP), and faces regulations for mandatory minimal treatment levels, and options for disposal of the treated wastewater (e.g., a dry riverbed, a flowing river, the ocean, or a nearby agricultural district).[1] The objective of the urban consumer is to maximize her utility from direct water consumption and available income, where the latter is a function of wastewater treatment quantity and effluent quality.

The region also includes an agricultural sector, represented by one grower of a certain crop.[2] The objective of the agricultural sector is to maximize net revenue, which is defined as the proceeds from sales of yield, minus production costs (including groundwater pumping cost). Finally, the third part of the regional model includes a common pool resource (CPR) in the form of an aquifer, the water of which supplies both the city and the agricultural irrigation demands. As a CPR, the aquifer is subject to impacts (quantity and quality) of return flows from irrigation water by the farming sector, and a congestion externality in the form of a lower water table, which is represented by increased pumping cost. We turn now to introduce the different model components.

The City

The city produces sewage as a share β (where $0 < \beta < 1$) of its freshwater consumption. We assume that this share is constant throughout time, and that the produced sewage needs to be treated and disposed of by the city, meeting certain disposal regulations. The city can select the disposal site, as long as it meets the disposal regulations. Therefore, the disposal location for the treated wastewater is an endogenous decision for the city. Different effluent disposal options imply different quality requirements of the treatment facility, as well as costs and benefits to the city and the entire region's economy. For generality purposes, we consider three options of effluent disposal sites in our framework. The first is the 'zero alternative' (alternative A), which is the default option available for the city. It implies that the city discharges the treated wastewater to a nearby water body (e.g., a dry riverbed, or a flowing river for an inland city, and the ocean for a coastal city). This is at a negligible cost, but with an opportunity cost represented by an environmental pollution damage function (to be depicted later). The second option (alternative B), is to remotely discharge the effluent so that the environmental pollution can be decreased, but an investment in conveyance infrastructure is necessary (e.g.,

[1] While we focus on the agricultural sector, the model can be adjusted such that the alternative for disposal would be a golf course, municipal irrigation, natural habitat and other sites.

[2] This assumption can be easily changed (without changing the nature of the problem) to include several growers and several crops. Adding such extension to the model will make the farming sector less sensitive to water quantity and quality, and will make the solution more flexible.

for an inland city, this would be building infrastructure to carry the effluent to a remote designated site, based on ecological concerns. Whereas for a coastal city, this would be building infrastructure for the safe discharge of the effluent further into the ocean). In any case, alternative B is characterized by additional significant conveyance infrastructure. The third and final option (alternative C) is to convey the effluent to a nearby agricultural district where, again, conveyance infrastructure is required, but differs from alternative B described above. In this final option the effluent is no longer considered a pollutant, but it becomes an intermediate input in agricultural production.

The city receives a steady supply of given quality of surface water from a source outside the region, and can also withdraw water from an underground aquifer, which is shared with the farming sector. The city's aggregated utility is assumed to be $U_t\left\{I\left(\beta\cdot(Q^u(t)+S(t));\varphi(t)\right),Q^u(t)+S(t)\right\}$, where $Q^u(t)$ and $S(t)$ are the quantities conveyed to and consumed by the city, at time t, from the groundwater aquifer and the outside source (which is constrained at a fixed level $\overline{S}$ over time), respectively, and $\varphi(t)$ is the quality level of the effluent coming out of the WWTP, which is subject to regulation and must therefore meet a predetermined required level $\overline{\varphi}$.[3] The function $I(\cdot)$ represents the available income spent by the aggregated consumer to purchase a composite good. It is composed of two arguments—the first is the amount of sewage produced (and treated) in the city $\beta\cdot(Q^u(t)+S(t))$, and the second is the effluent quality $\varphi(t)$. It is assumed that $I_1<0, I_{11}>0; I_2>0, I_{22}<0$; where I_i and I_{ii} stand for the first and second partial derivatives with respect to the *i*th argument, $i=1,2$. Like Feinerman et al. (2001), we assume that the cost of treating the sewage produced in the city is embedded within the available income function $I(\cdot)$, and therefore affects the level of aggregate utility, as the city is assumed to own the WWTP. Therefore, as more water is consumed by the city (and more sewage is

[3] Effluent quality should obviously be considered as a vector of quality components (e.g., salinity, nutrients, BOD, COD, SS, phosphorus, and others), however for simplicity, we assume that $\varphi(t)$ represents only one quality component (say salinity) for the convenience of presentation. We should also clarify that the lower the level of $\varphi(t)$, the better the quality, and the hypothetical highest quality would be $\varphi(t)=0$. It is also important to note that while certain contaminants are subject to regulation (e.g., BOD, COD and SS), salinity is not and therefore its level could become a source for negotiation between the farming sector and the city.

produced and needs to be treated), the income available for other goods beyond water is reduced; however, this effect diminishes due to the WWTP economies of scale.

With respect to quality, the poorer the quality of effluent produced (meaning higher $\varphi(t)$), the cheaper the treatment cost, which increases available income. As in the case of quantities, we assume that the marginal effect of effluent quality diminishes as $\varphi(t)$ rises, hence the second derivative of $I(\cdot)$ with respect to $\varphi(t)$ is negative. We also assume that the utility function $U(\cdot)$ is well-behaved (hereon after the time dependency may be omitted in several places in the paper, due to presentation convenience considerations), therefore $U'(\cdot)>0, U''(\cdot)<0$; with respect to both available income and quantity of water consumed.[4] Quality standards are assumed to be imposed on the water supplied to the city from all sources. It is therefore reasonable to assume that the outside water source quality always adheres to these standards, and therefore it is not explicitly represented. However, as quality of groundwater deteriorates, it is safe to assume that it becomes less appealing for city needs. Specifically, we assume that once groundwater quality $g(t)$ exceeds the level imposed by regulation $\bar{g}$, it needs to be treated at cost $h(g(t))$ in order to be supplied to the city, where $h(\cdot)$ is continues and twice differentiable, such that $h'(\cdot)>0$ and $h''(\cdot)>0$ for $g(t)\geq\bar{g}$, otherwise $h(g(t))=0$.

As previously mentioned, the treated wastewater needs to be conveyed to a chosen destination, where each alternative (i.e., A, B or C) implies different cost and benefit considerations for the region. We symbolize $E(t)$ as the amount of effluent being discharged to the environment in all the alternatives, such that

(1) $E(t)=\beta\cdot(Q^{u}(t)+S(t))-O(t)-A(t)$

Where $O(t)$ and $A(t)$ are the quantities of effluent conveyed to the ocean (under alternative B) or to the agricultural district (under alternative C), respectively. As noted above, we further assume that discharging treated wastewater to the environment is associated with a

[4] Other standard utility characteristics for the existence of internal solution are assumed, i.e., let Q be set equal to $Q^{u}+S$ then it is assumed that $\lim_{Q\to 0} dU/dQ \to \infty$; $\lim_{Q\to\infty} dU/dQ \to 0$; $dU/dQ \geq 0\ \forall Q\geq 0$.

social cost in the form of a damage function $D(E(t),\varphi(t))$.[5] We follow Farrow et al. (2005) by assuming that the damage function is linear with respect to both arguments.[6,7] The other two alternatives for effluent disposal (i.e., B and C) require investments in conveyance infrastructure, and are characterized by increasing marginal cost curves of conveyance. These variable cost curves are depicted by $\nu(O(t))$ and $\upsilon(A(t))$ for the ocean (B) and agricultural irrigation (C) alternatives, respectively. Both functions exhibit the following similar characteristics:

(2) $\nu'(\cdot)=\upsilon'(\cdot)>0 \quad \forall O(t)=A(t)$

(3) $\nu''(\cdot)=\upsilon''(\cdot)>0 \quad \forall O(t)=A(t)$

The underlying assumption is that once a decision to build a conveyance infrastructure to either location has been made, it bears a fixed cost (which is the amortized cost of investment, and is notated by ν_0, and υ_0 for alternative B, and C, respectively), and that conveyance of greater quantities of water has an increasing marginal cost.[8] It is also assumed that $\nu_0 > \upsilon_0$, accounting for the difference in investment levels associated with each option (we consider the ocean disposal option to be more remote and therefore it necessitates higher fixed costs). Furthermore, we assume that effluent diverted to the farming sector for irrigation (since it is considered an intermediate input in agricultural production) could be sold to farmers by the city at a price $P^E(t) \geq 0$, which could potentially be subject to a negotiation process.

Finally, each unit of freshwater supplied from the aquifer bears the cost of extraction $C(G(t))$, which is decreasing and convex in the groundwater stock level, G, in each period t

[5] It can be argued that although effluent quality is regulated and monitored, some contaminants like pharmaceuticals, nitrogen, and phosphorous are found in higher levels in treated effluent than in other water sources, and therefore are posing health and environmental risks (Hernando et al., 2006).

[6] Specifically, we assume that $D(0,\varphi(t))=0$ and $D(E(t),0)=0$.

[7] Horan (2001) argues in favor of a non-decreasing convex functional form of the damage associated with water quality pollution. We choose the linearity assumption on generality considerations, and discuss the implications of each of these assumptions in detail in Appendix A.

[8] We also assume that conveyance infrastructure capacity level, once decided upon, is sufficient for the system's existing and future needs i.e., it is exogenous, such that just the timing of building it becomes a decision, and the level of capacity itself can be disregarded as a constraint or a decision in the optimization model.

. We also assume that this unit cost approaches zero as stock reaches its maximal level $\overline{G}$ (i.e., $\lim_{G \to \overline{G}} C(G(t)) \to 0$). Without loss of generality, we assume that surface water supply is costless.

The Farming Sector

The farming sector grows one crop that is sensitive to both water quantity and its quality. The farm sector is a price taker and receives a payment of $P^Y(t)$ per unit of output sold at the market. Its water sources are the groundwater aquifer, which is shared with the city, treated wastewater from the WWTP, and precipitation.[9] The per-unit area production function of the crop is $Y_t\{w(t), \psi(g(t), \varphi(t)); r(t)\}$, where $w(t)$ and $r(t)$ are the per-unit area applied water and precipitation levels, respectively. Let $\psi(\cdot)$ be the quality of the applied water, which is a function of water qualities from the different sources that are applied for irrigation, and let it be positively and linearly correlated with each source independently (i.e., $\psi_g, \psi_\varphi > 0$; $\psi_{gg}, \psi_{\varphi\varphi} = 0$). We avoid assigning $\psi(\cdot)$ with a specific functional form at this stage, and leave it for the illustration section.[10] We assume also that production is increasing, both as water quantity per unit of land increases, and as water of higher quality is applied.[11] Both effects diminish with rising quantities and qualities. We also assume that there is a non-zero elasticity of substitution between water quantity and quality, and between applied water and precipitation. These assumptions are summarized below, in a notational form. Where Y_i and Y_{ii} again denote the first and second partial derivatives with respect to the *i*th argument, this time $i = 1,2,3$.

(4) $Y_w > 0, Y_{ww} < 0; Y_\psi < 0, Y_{\psi\psi} > 0;$

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[9] One could also consider that the farming sector has a surface water source in the same way the city does; however, since competition over groundwater between these sectors is already accounted for, and since stochasticity is currently ignored, including a surface water source for agriculture becomes redundant.

[10] For simplicity, one can assume that the quality of water applied is composed of a constant (representing the use of contaminating inputs, such as fertilizer and pesticides, the introduction of which through drip irrigation is becoming a common practice in modern agriculture), plus a weighted average of the qualities according to water consumption from the different sources. Notice though, that such assumption imposes some other characteristics of $\psi(\cdot)$; we discuss these in Appendix A.

[11] Similar to the utility function of the city, we assume internal solution properties for the per-unit land agricultural production function, with respect to water applied and its quality (see footnote 4).

(5) $\frac{Y_w}{Y_r}, \frac{Y_w}{Y_\psi} \neq 0$;

Assume for simplicity that the farming sector is not limited in its cultivable land and labor force. The only constraint the farming sector faces is water quantity. The costs associated with agricultural production are the costs of the water inputs, i.e., $C(G(t))$ for the groundwater supply $Q^a(t)$, and the price of treated wastewater $P^E(t)$ for the effluent diverted from the WWTP, $A(t)$; and other costs associated with the production process (such as labor, fertilizer, and others), which are expressed as a function $f(X(t))$ of the cultivable land $X(t)$.

The Groundwater CPR

As mentioned above, the groundwater source is represented by two states: water stock level $G(t)$, and quality level $g(t)$. The equations of motion defining the states at each period t are as follows:

(6) $\dot{G} = R(t) + \theta \cdot (X(t) \cdot w(t)) - Q^u(t) - Q^a(t)$

(7) $\dot{g} = e(\psi(t)) - \delta(G(t)) \cdot g(t)$

It is assumed that the groundwater table is subject to recharge volume $R(t)$, which originates from rainfall and from deep percolations by the farming sector at a constant rate θ per unit of applied water, where $0 < \theta < 1$, and water extractions for the use of both the city and the farming sector. As common in standard optimal control problems of renewable resources exploitation (e.g., Burt, 1964; Cummings and Winkelman, 1970; Tsur and Graham-Tomasi, 1991; among others), groundwater stock level is depleted over time, implying that $\dot{G} < 0$ until a steady-state is reached, at which time $\dot{G}$=0.

Conceptually, we follow Roseta-Palma (2002, 2003) to describe the evolvement of water quality in the aquifer. We assume that contaminants dissolve naturally in the ground at a given rate $\delta(G(t)) > 0$. It is also assumed that this decay rate is higher when groundwater stock level rises, however this effect diminishes (i.e., $\delta'(G(t)) > 0$; $\delta''(G(t)) < 0$). The function $e(\psi(t))$, which is the first component in equation (7) accounts for contamination caused by agricultural activity (which is assumed to be always positive), and is defined as a function of the water quality applied.[12] It is presumed that irrigating with water high in contaminant levels increases the rate of quality degradation in the aquifer (Candela et al., 2007; Katz et al., 2009). Specifically, we assume a non-decreasing convex function for $e(\psi(t))$. It then follows that $e'(\psi(t)) > 0; e''(\psi(t)) > 0$.

All the three components described above are integrated now into one regional model (Figure 1), which then is solved for social welfare maximization.

The Regional Model

We first solve the social planner optimization problem for the entire region, postponing the discussion regarding private solutions to the next section. We assume that alternative B is already in place, meaning that investment in conveyance infrastructure to carry effluent to the ocean had already occurred in the past, and is now a sunk cost. This assumption requires some justification. We use Appendix A to depict the social planner welfare maximization problems (A1, A2, and A3) associated with each of the different disposal alternatives (A, B and C) separately, assuming for each, that existing conditions prevail for an infinite horizon, and ignoring the different investment options, their optimal timing, or sequence. We denote by Z_i the solution space for problem i where $i \in \{\text{A1}, \text{A2}, \text{A3}\}$, and z_i^* as the optimal solution for a given empirical setting of problem i, such that $z_i^* \in Z_i$ and $V_i(z_i^*)$ is the maximum value of the objective function, given the optimal solution of the problem z_i^*.

Proposition 1. *Facing identical functional forms and sets of parameters*

$$V_{\text{A3}}(z_{\text{A3}}^*) \geq V_{\text{A2}}(z_{\text{A2}}^*) \geq V_{\text{A1}}(z_{\text{A1}}^*)$$

The proof is provided in Appendix A. Even under Proposition 1, it can be argued that solving alternatives A and B sequentially can be informative in terms of deciding if the incremental benefits exceed the costs of investment, and finding the optimal timing for switching between them. However, if we assume that there is such an optimal timing, then

[12] Roseta-Palma (2002, 2003) takes a more general approach and explicitly includes the use of contaminating inputs in agriculture to account for their effect on groundwater quality. We accommodate this approach by implicitly incorporating it within the applied water quality function (see footnote 10). Since our approach focuses on the role of treated wastewater reuse in economic tradeoffs among competing sectors over water allocations, we find this solution to be a better fit for the scope of our work.

there is no loss in generality from the social planner perspective, and therefore no added value for our discussion in following the sequential approach. This, however, might not be the case when private solutions are analyzed, and we would therefore return to that point later in our discussion about sectoral optimization problems and their solutions. Another aspect, and probably a more important one, is the choice of whether and when to invest in either or both alternatives B and C. We introduce some further notation. Let F_j be the investment associated with alternative j, where $j \in \{B, C\}$, and let T_j^* be the optimal timing for investing in that alternative.

Proposition 2. *Let alternative A set the initial conditions for a regional social welfare planner facing the two alternatives for effluent discharge B and C. Then,*

$$if \;\; F_B > F_C \Rightarrow T_C^* < T_B^* \Rightarrow T_B^* \in \{\varnothing\}$$

The proof is provided in Appendix A. According to Proposition 2, it immediately follows that if one wants to account for alternative B as a possible option in the social planner's solution, it must be assumed to be the initial setup.

Let us now define $V^{\tau_C}\left(z^*_{\tau_C}, G(T_C), g(T_C)\right)$ as the value function corresponding to the optimal solution $z^*_{\tau_C}$, over the period $\tau_C \equiv \{T_C, \infty\}$ and given the initial conditions $G(T_C)$ and $g(T_C)$, which are all quantity and quality possible states at time T_c. Following the above, the regional social planner's problem (8) is presented below.

$$\text{(8)} \quad \underset{Q^u, S, Q^a, A, O, \phi, X, w, T_C}{Max} \int_0^{T_C} e^{-rt} \cdot \Big[U\left\{ I\left(\beta \cdot (Q^u(t) + S(t)), \varphi(t)\right), Q^u(t) + S(t) \right\}$$
$$+ P^Y(t) \cdot X(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} - f(X(t)) - C(G(t)) \cdot (Q^u(t) + Q^a(t))$$
$$- h(g(t)) \cdot Q^u(t) - v(O(t)) - D(E(t), \varphi(t)) \Big] dt + e^{-rT_c} \cdot \left(V^{\tau_C}_{A3}(z^*_{A3\tau_C}) - F_c \right)$$

s.t.

a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$

b) $\dot{g} = e(\psi(t)) - \delta(G(t)) \cdot g(t)$

c) $\varphi(\text{t}) \le \overline{\varphi}$

d) $X(t) \cdot w(t) \leq Q^{a}(t) + A(t)$

e) $S(t) \leq \bar{S}$

f) $A(t) + O(t) \leq \beta \cdot (Q^{u}(t) + S(t))$

g) $G(0) = G_0$

h) $g(0) = g_0$

The solution to this problem relies on a two step procedure. First, $V^{\tau_c}\left(z^{*}_{\tau_c}, G(T_C), g(T_C)\right)$ needs to be characterized for any given initial conditions $G(T_C)$ and $g(T_C)$, and for every possible timing T_c. The second step is then maximizing the integer in problem (8) between time zero to T_c, with T_c being a decision variable in the optimization, and taking $V^{\tau_c}\left(z^{*}_{\tau_c}, G(T_C), g(T_C)\right)$ as a boundry value. The properties of the latter obviously affect the transversality conditions for the optimal solution, as discussed in detail later on. The characterization of the boundry value depicted above is equivalent to solving problem A3 (Appendix A) for changing initial conditions of the states $G(T_C)$ and $g(T_C)$, and sub-periods τ_c. It is important to notice that decisions about $A(t)$, and about $\varphi(\mathrm{t})$ as a component in the crop production function, only become relevant after (and if) investment in conveyance infrastructure to the agricultural sector at time T_c is decided upon (for example, $A(t)$ is omitted from constraints (d) and (f) prior to such an investment). We define for problem (8) above its respective Lagrangian function, and derive the first-order conditions (FOC)—these are rearranged and presented below in equations (8.1)- (8.14). We also denote by $\lambda_{\varphi}(t)$, $\lambda_{W}(t)$, $\lambda_{S}(t)$, and $\lambda_{E}(t)$ the Lagrangian multipliers (shadow values) associated with constraints (c) through (f), respectively. $m_1(t)$ and $m_2(t)$ are the co-states for the equations of motion in constraints (a) and (b), respectively.

(8.1) $\lambda_{W}(t) = C\big(G(t)\big) + m_1(t)$

(8.2) $$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C\big(G(t)\big) + h\big(g(t)\big) + \beta \cdot \big(D_E - \lambda_E(t)\big) + m_1(t)$$

(8.3) $$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot \left(D_E - \lambda_E(t)\right) + \lambda_S(t)$$

(8.4) $$\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_\varphi}_{\text{VMP of Applied Water Quality in Agriculture (-)}} - m_2(t) \cdot \underbrace{e_\psi \psi_\varphi}_{\text{Marginal Contamination Rate (+)}}$$

(8.5) $$P^Y(t) \cdot Y\{w(t), \psi(g(t), \varphi(t)); r(t)\} = f'(X(t)) + \left(\lambda_W(t) - \theta \cdot m_1(t)\right) \cdot w(t)$$

(8.6) $$X(t) \cdot \left(P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)\right) = 0$$

(8.7) $$\dot{m}_1 - r \cdot m_1(t) = \underbrace{C'(G(t))}_{\text{Marginal Cost of Extraction (-)}} \cdot \left(Q^u(t) + Q^a(t)\right) + m_2(t) \cdot \underbrace{\delta'((G(t))}_{\text{Marginal Decay Rate (+)}} \cdot g(t)$$

(8.8) $$\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\text{Marginal Contamination Rate (+)}} - \delta(G(t)) - r\right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\text{Marginal Costs of Groundwater Treatment for Urban Consumption (+)}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\text{VMP of Applied Water Quality in Agriculture (-)}}$$

(8.9) $$\lambda_\varphi(t) \cdot \left(\overline{\varphi} - \varphi(t)\right) = 0\,,\ \lambda_\varphi(t) \geq 0$$

(8.10) $$\lambda_W(t) \cdot \left(Q^a(t) + A(t) - X(t) \cdot w(t)\right) = 0\,,\ \lambda_W(t) \geq 0$$

(8.11) $$\lambda_S(t) \cdot \left(\overline{S} - S(t)\right) = 0\,,\ \lambda_S(t) \geq 0$$

(8.12) $$\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - O(t) - A(t)) = 0\,,\ \lambda_E(t) \geq 0$$

(8.13) $$\lambda_E(t) = D_E - v'(O(t))$$

(8.14) $$\lambda_E(t) = \lambda_W(t) + D_E - \upsilon'(A(t))$$

The following interpretations refer to the optimal solution. Equation (8.1) states that in the optimal solution the shadow value associated with the available water constraint for irrigation should be equal to the sum of the unit cost of extraction and the scarcity rent. Equations (8.2) and (8.3) equate the marginal utility of water consumption to the total (social) marginal cost associated with the use of each water source—groundwater and the outside source, respectively. As these sources are perfect substitutes, if the outside source is plentiful (i.e., $\overline{S}$ is very large and the associated availability constraint (e) is not binding, and therefore $\lambda_S = 0$), equations (8.2) and (8.3) imply that the city will only consume surface water. In (8.4) marginal utility from contamination (i.e., treatment of effluent to a lower quality) is equated with the social cost associated with it. It is worthwhile noting that in the private case of an unregulated contaminant (e.g., salinity), and prior to investment in conveyance capacity to the farming sector (i.e., $\psi_{\varphi} = 0$), the right-hand side of equation (8.4) will include only the marginal damage associated with higher contamination. This means that all other components, if included in the solution, will lead to a higher optimal treatment level. Placing equation (8.1) into (8.5) and (8.6), and given that there exists an internal solution with respect to cultivable land (i.e., $X^{*}(t) > 0 \;\forall t$) yields the following respectively:

$$(8.15) \quad P^{Y}(t)\cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + \left[\underbrace{C(G(t)) + (1-\theta)\cdot m_1(t)}_{\text{VMP of Water Applied}(+)}\right]\cdot w(t)$$

$$(8.16) \quad P^{Y}(t)\cdot Y_w = C(G(t)) + (1-\theta)\cdot m_1(t)$$

These two equations dictate that the value of marginal product in agriculture from both inputs (water and land) will be equal to their marginal cost, accounting for scarcity and deep percolation effects. Equations (8.7) and (8.8) define the optimal paths for the co-states associated with groundwater stock level, and quality, respectively. We express below these relationships in the form of growth rates:

$$(8.17) \quad \frac{\dot{m}_1}{m_1(t)} = r + \frac{\left[\overbrace{C'(G(t))\cdot(Q^{u}(t)+Q^{a}(t))}^{(-)} + \overbrace{m_2(t)\cdot\delta'((G(t))\cdot g(t)}^{(-)}\right]}{m_1(t)}$$

$$(8.18) \quad \frac{\dot{m}_2}{m_2(t)} = r + \delta(G(t)) - e_{\psi}\psi_g + \frac{\left[\overbrace{h'(g(t))\cdot Q^{u}(t)}^{(+)} - \overbrace{X(t)\cdot P^{Y}(t)\cdot Y_{\psi}\psi_g}^{(-)}\right]}{m_2(t)}$$

Recall that groundwater stock level G decreases with time, and pollutant level g increases (it is also important to notice at this stage that $m_2(t)$ should obviously be negative, as it is associated with g—which is a pollutant), where the latter occurs due to the positive difference between the farming sector's contaminating activity and the aquifer's decreasing resilience to pollution as the water table declines. Note that scarcity is affected, both by the increasing cost of extraction, but also by the increasing value of higher quality water (Equation (8.17)). The shadow value of quality (Equation (8.18)) accommodates the net rate of pollutant accumulation in the aquifer, and also the effect on the value of production. Equations (8.9) through (8.12) are the usual Karush-Kuhn-Tucker conditions and dictate that the shadow value of any binding constraint in the optimal solution must be non-negative. The last two equations determine the value of $\lambda_E(t)$—the shadow value associated with the effluent availability constraint, and are derived from the decision whether to allocate effluent to either (or both) of the disposal alternatives B and C.

A steady state arises when the time derivatives of the states and co-states are set at zero, which in turn translates into the following:

(9) $$Q^u(t) + Q^a(t) = R(t) + \theta \cdot X(t) \cdot w(t)$$

(10) $$g(t) = \frac{e(\psi(t))}{\delta(G(t))}$$

(11) $$m_1(t) = -\frac{\left[C'(G(t)) \cdot \left(Q^u(t) + Q^a(t)\right) + m_2(t) \cdot \delta'((G(t)) \cdot g(t)\right]}{r}$$

(12) $$m_2(t) = \frac{\left[h'(g(t)) \cdot Q^u(t) - X(t) \cdot P^Y(t) \cdot Y_\psi \psi_g\right]}{e_\psi \psi_g - \delta(G(t)) - r}$$

As is common in these type of optimal control problems, according to the first equation, steady state extraction would be equal to the level of recharge (including deep percolation that originates from irrigation). The second equation implies that contaminant level would be set according to the ratio between the level of agricultural contamination, and the pollutant decay rate in the aquifer. According to equation (11), scarcity rent will be higher at lower levels of groundwater stock, and as quality degrades. Groundwater quality shadow value is larger when contamination impacts (e.g., salinity) on agricultural production are higher.[13] It is also noteworthy to explain that the negativity of this co-state rests on the assumption that the difference between the marginal contamination rate and the aquifer's decay rate is not too large (specifically, for this assumption to hold, that difference cannot exceed the discount rate r). In other words, the agriculture contamination function $e(\psi(t))$ is expanding at a moderate and constrained rate.

[13] There is also an effect that stems from the increase in groundwater treatment cost to meet the requirements associated with water quality that is supplied to the city; however, as explained in Appendix A, in the social planner's problem this cost is irrelevant.

Finding the optimal timing T_c for investing in conveyance infrastructure connecting the WWTP with the farming sector involves the derivation of the transversality condition developed by Hartwick et al. (1986), which requires that the net benefits stemming from water allocations after the timing of investment would exceed the net benefits obtained prior to that investment, at least by the interest payment for the investment (Holland and Moore, 2003). Holland and Moore (2003) rely on a formal proof developed by Holland (2003) to identify a continues price path (which translates into a continues consumption path, as well). This, in turn, enables the derivation of an optimal time rule for investing in a water import project, in which the original supply alternative is a renewable groundwater aquifer. We argue that the problem presented in our paper is no different for the purposes of satisfying the same derivation, and we therefore avoid the burdensome description associated with it. Our argument can be easily illustrated using a quantity allocation optimization problem. For the sake of the illustration only, and without losing generality, it helps to consider a simplistic static example, which we depict in Figure 2.

Holland and Moore (2003) consider a water import project to augment a renewable groundwater source. Let's notate by $\overline{Q}$ the quantity-constraining extractions prior to investing in the import project, and by $\bar{I}$ the capacity of the new project. On the demand side, let's consider the case at hand, in which supplies need not just satisfy a single sector, but two—a city, and a farming sector, both represented by demand curves that are derived from a utility function, and the value of production function, respectively. Let c_G be the cost of extraction from the aquifer and c_I the unit cost of imported water (where we assume that $c_I < c_G$, but could have equally illustrated the same for the opposite case). Let Q_0^u and Q_0^a be the optimal quantities consumed in the city and by the farming sector, respectively, from both sources. The '0' notation is used for original quantities (prior to the import project), and the notation '1' (instead) for the quantities consumed after the project's implementation. In Figure 2, the original (optimal) allocation is dictated by the following:

(13) $Q_0^u + Q_0^a = \overline{Q}$

(14) $D_U\left(Q_0^u\right) = D_A\left(Q_0^a\right) = c_G + \mu_0$

where D_U and D_A are the demand curves for the city and farming sector, respectively, and μ_0 is the shadow value of the binding constraint (13).

After the introduction of the import project, the supply constraint becomes $\overline{Q} + \bar{I}$, such that now the new optimal quantities comply with:

(15) $Q_1^u + Q_1^a = \overline{Q} + \bar{I}$

(16) $D_U\left(Q_1^u\right) = D_A\left(Q_1^a\right) = c_G + \mu_1$

where μ_1 now represents the shadow value of the constraint in (15). As illustrated, let the optimal allocation dictate that the import project is used to full capacity and its water conveyed to the city, and specifically that $Q_0^u = \bar{I}$. It follows that $Q_1^u + (Q_1^a - Q_0^a) = \overline{Q}$. Returning to our original notation and requiring that $C(G)$ will be constant and equal to c_G and that $\upsilon'(A)$ be also constant and equal to c_I. Since water treated and conveyed to the farming sector from the city are equal to $\beta \cdot Q_1^u$, if we set β to be equal to $\frac{Q_0^u}{Q_1^u}$, then it immediately follows that under the same framework presented in Figure 2, the solution for both settings would be identical, with the only difference being the income distribution between the city and the agricultural sector.

In terms of quality, as discussed in detail in Appendix A, the necessary optimality conditions facilitate a solution in which the quality of the effluent will be equal to the quality of groundwater, therefore any differences associated with this dimension would be mainly manifested by changes in shadow values, and should not rebut our main argument.

2.2 Private Solutions

As previously mentioned, the optimal private solution of each sector might differ from the social planner one. The following discussion is not aimed at portraying these differences to their full extent, nor is it aimed at depicting the mechanisms necessary to move the competitive private solutions closer to the 'first best' social planner's solution presented above. These are challenges that are well documented in previous work (Koundouri et al. 2017), and are beyond the scope of our paper. Relying on our description of the social planner's problem and the different model components in previous sections, we introduce below each sector optimization problem, and briefly discuss its potential optimal solution.

We start by describing the farming sector optimization problem. As previously mentioned, the objective of the farming sector is to maximize its discounted annual net revenues, where the sector's annual net revenue is defined as follows:

$$(17)\quad P^{Y}(t)\cdot X(t)\cdot Y\{w(t),\psi(g(t),\varphi(t));r(t)\}-f(X(t))-C(G(t))\cdot Q^{a}(t)-P^{E}(t)\cdot A(t)$$

The solution set is confined by constraints (a), (b), (d), and (f), taken from the social planner optimization problem (8) described above. In these constraints, and for the entire optimization problem of the farming sector, the city's decisions are taken as given constants. The farming sector is left to choose the area of cultivable land $X(t)$, the amount of water applied to the crop per unit of land $w(t)$, and the quantities consumed from each water source, groundwater $Q^{a}(t)$ and treated wastewater $A(t)$, respectively. Obviously in the case that one source is plentiful and dominates the other in terms of costs (cheaper) and quality (less contaminated), the optimal solution would suggest utilizing that source only. However, as broadly studied and demonstrated, blending could become an optimal strategy, depending on the empirical settings of the problem (Feinerman and Yaron, 1983; Knapp and Dinar, 1984; Dinar et al., 1986; Kan et al., 2002; Kan, 2008; and Kan and Rapaport-Rom, 2012).

The city's objective is to maximize its discounted net benefits, which are defined as the aggregated utility (in monetary terms) from water consumption and income spent on a composite good, and from sales of effluent to the farming sector, minus the costs of water extraction, discharge taxes (which are the realization of the damage function introduced in the social planner's problem), conveyance to (alternative) disposal sites, and interest rate paid over the investment in conveyance infrastructure. The annual net benefits for the city are depicted below:

$$(18)\quad U\{I(\beta\cdot(Q^{u}(t)+S(t)),\varphi(t)),Q^{u}(t)+S(t)\}+P^{E}(t)\cdot A(P^{E}(t),\varphi(t))-(C(G(t))+h(g(t)))\cdot Q^{u}(t)$$
$$-\nu(O(t))-\upsilon(A(P^{E}(t),\varphi(t)))-\phi(E(t),\varphi(t))$$

The choice variables for the city in its optimization problem are the quantities consumed from both sources, groundwater $Q^{u}(t)$ and surface water $S(t)$, respectively, effluent quality $\varphi(t)$, and the amount of effluent disposed to the ocean $O(t)$. Contrary to the social planner, the city can only choose the price $P^E(t)$ at which effluent could be sold to the farming sector. The quantity that will be demanded, conveyed to, and consumed by the farming sector is therefore a function of both that price, and the quality of the treated wastewater $A\left(P^E(t),\varphi(t)\right)$. This demand function for treated wastewater by the farming sector is downward slopping and convex in both arguments, representing the characteristics of the yield response function $Y\{\cdot\}$ presented above. The function $\phi(\cdot)$ represents a pollution tax that would be imposed on the city, due to discharges to the environment, is a monetary realization of the damage function $D(\cdot)$ presented above, and therefore possesses identical characteristics. Notice that Equation (18) is formalized such that investment in conveyance infrastructure to both disposal alternatives (B and C), had already occurred, and includes the variable conveyance cost functions $\nu(\cdot)$ and $\upsilon(\cdot)$, disregarding the interest payment associated with both options. This is obviously not the initial setting for the city's optimization problem. When depicted in full, the formal optimization problem should accommodate the choice of whether to invest in either B or C, or in both, and the decision about the sequence of events (i.e., first invest in B and afterwards in C, or the opposite, or invest only in one of the alternatives). The discounted Equation (18) is maximized subject to constraints (a), (b), (c), (e), and (f) taken from problem (8), to yield the optimal paths of states and controls for the city. Where the quantity $A(t)$ in constraint (f) is substituted with the demand function $A\left(P^E(t),\varphi(t)\right)$, and all variables that are beyond the control of the city are taken as given constants in the optimization process.

An interesting point that is relevant for our discussion is the decision about the disposal location, its associated investment, and the sequence of events from the city's perspective. Different from the social planner point of view, the city's private problem does not account for possible externalities resulting from irrigating with lower- or higher-quality treated wastewater, or the effects of extraction decisions on the costs and benefits accruing to the farming sector due to depletion of the groundwater stock. Furthermore, as the quantity of effluent demanded by the farming sector is a function of $\varphi(t)$ and the price $P^E(t)$, there is a possibility under alternative C that the optimal solution will also include a positive level of taxes that are imposed on the city, due to the fact that not all effluent is consumed by the farming sector. The resulting overall argument is that Propositions 1 and 2 do not necessarily hold for the private problem of the city.

To address this argument, we examine the conditions that need to prevail, such that for each alternative j, where $j \in \{B, C\}$, there would be an optimal timing $T_j^* \in \{0, \infty\}$ for investment, with respect to alternative n, where $n \in \{A, B, C\}$ and $n \neq j$. We denote the set of optimal solutions z_{nj}^*, satisfying these conditions as $Z_{j \succ n}$, and we say that $\{z_{nj}^* \in Z_{j \succ n} \text{ if } \chi_{nj} \geq 0\}$; where $\chi_{nj} = V_{nj}(z_{nj}^*) - V_n(z_n^*)$. Following earlier definitions, $V_n(z_n^*)$ is the maximum value of the objective function, given the optimal solution of the problem, z_n^*, assuming that alternative n prevails for an infinite horizon, whereas $V_{nj}(z_{nj}^*)$ is the maximum value of the objective function, given the optimal solution of the problem, z_{nj}^*, and where investment at alternative j had occurred at optimal time T_j^*. Specifically, for the case of investing in alternative C with respect to alternative A, χ_{AC} can be broken down to effluent sales Ω_{AC} (which is just $P^E(t) \cdot A\left(P^E(t), \varphi(t)\right)$, where $\varphi(t)$ and $P^E(t)$ are at their optimal levels), and $v_{AC}(z_{AC}^*)$, which stands for all other changes in the objective function components associated with moving from solution z_A^* to z_{AC}^*. For alternative C to be superior to alternative B, it must hold that $\chi_{BC} \geq 0$, and therefore it also must hold that $\chi_{AC} \geq \chi_{AB} \;\; \forall T_B^* \in \{0, \infty\}$. Using the definitions above, the last statement can be written as:

(19) $\Omega_{AC} + v_{AC}(z_{AC}^*) \geq \chi_{AB}$.

Since for any positive T_B^* it must hold that $\chi_{AB} \geq 0$, then it follows that the set of solutions z_{BC}^* satisfying (19), is only a subset within $Z_{C \succ A}$, which implies that there exists a subset of optimal solutions $\{z_{AC}^* \in \tilde{Z}_{C \succ A}\}$, such that $\{\tilde{Z}_{C \succ A} \in Z_{C \succ A}\}$, however that $\{\tilde{Z}_{C \succ A} \notin Z_{C \succ B}\}$. It immediately follows that Propositions 1 and 2 cannot hold for the private optimization problem of the city.

We now turn to illustrate our conceptual findings. We use data and functional forms taken from existing literature, and without focusing on a specific region.

3. Illustrative Example

We begin our illustration by portraying the two water-consuming sectors in the region's economy—a city, and an agricultural district. For the first, we choose to represent the utility function in the common Cobb-Douglas functional form as depicted in equation (20).[14]

(20) $$D_u \cdot Q_u^{\eta_1+1} \cdot \varphi^{\eta_2\cdot(\eta_3+1)}$$

Where η_1 is the inverse price elasticity, h_3 is the inverse income elasticity, and h_2 is the effect of changes in effluent quality level φ on income, such that φ^{η_2} represents the available income function $I(\cdot)$ described earlier; D_u is a scaling parameter that is added to maintain consistency in units used, and also facilitates annual income and population growth rate trends, which we assume to be 2.88% and 1.24%, respectively (World Bank, n.d.). For both price and income elasticities we use a range of estimates taken from the literature (Espey et al., 1997; and Dalhuisen et al., 2003). For the farming sector, we use calibrated production functions from Kan (2003) for two alternative crops—cotton and tomatoes, which differ at their level of salinity tolerance (Maas and Hoffman, 1977). The general form is depicted in Equations (21) and (22) below.

(21) $$Y = b_1 \cdot (ev - \underline{ev}) + b_2 \cdot (ev - \underline{ev})^2$$

where,

(22) $$ev = \frac{\overline{ev}}{1 + \alpha_1 \cdot (\psi + \alpha_2 \cdot w^{\alpha_3})^{\alpha_4}}$$

Where ev (feet/year) represents periodical evapotranspiration, $\overline{ev}$ is the maximum available evapotranspiration, and $\underline{ev}$ is the minimal evapotranspiration required for crop production; w and ψ are as defined above, and α_1 through α_4, b_1 and b_2 are scalars, with $b_1 \geq 0$, $b_2 \leq 0$. The set of parameters for both sectors' functions are presented in Table 1, as well as crop prices that are taken from Kan (2003), crop production-cost function parameters (f_0

[14] While this representation is a very simplified version of the general utility function presented in the conceptual part of the paper, it still carries all the necessary qualitative characteristics assumed.

and f_1)—calibrated based on observed average size farm in the US (USDA, 2017), and per unit of land observed water applications ($\tilde{w}$)—taken from Johnson and Cody (2015).

For the groundwater source, we choose again to avoid focusing on a specific basin, and adopt the characteristics of the aquifer presented in Roseta-Palma (2003).[15] As suggested by the author, as a possible extension of her approach, we introduce dependency between groundwater storage level and the decay rate, according to the following relationship $\delta_0 + \delta_1 \cdot G^{\rho}$, where δ_0 and δ_1 are positive scalars, and $0 < \rho < 1$.[16] The other component in the groundwater quality equation of motion (constraint (b) in Problem (8)) is depicted by $e \cdot \psi^2$, where e is a positive scalar, and ψ, the quality of applied water in agriculture is defined according to Equation (23) as suggested earlier (footnote 10).

(23) $$\psi = \gamma + \frac{Q_a \cdot g + A \cdot \varphi}{Q_a + A}$$

Where γ stands for salinity induced by using contaminating inputs (such as fertilizer and pesticides) in crop production; the rest of the elements in Equation (23) are decision and state variables of the optimal control problem (8) described above. The different parameters described above controlling the groundwater quantity-quality states are presented in Table 2.

3.1. Results

Using the GAMS platform, we solve the empirical application as it is described above, and refer to this solution as the base-scenario (using the original parameters in Tables 1 and 2). We choose a long-enough planning horizon to ensure convergence towards a steady-state. Figures 3 and 4 depict the outcomes of that base scenario with respect to the two alternative crops indicated above—cotton (C), and tomatoes, marked (T).[17]

We found that for this base scenario all effluent is diverted towards agriculture. We perform several sensitivity analyses (to be described in detail below), and while in certain cases

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[15] Two noteworthy modifications with respect to Roseta-Palma's illustration are (1) units, which we changed from metric to imperial/United States Customary System; and (2) while Roseta-Palma (2003) used pumping lift as the state variable in her illustration, we are keeping the illustration in storage level measures—the transition between the two is straightforward, and relies on basic hydrologic principals (Heath, 1983, p. 28).

[16] For the calibration of this function, we require more information than the fixed decay rate reported by Roseta-Palma (2003). Unfortunately we couldn't find a reference for that information, and therefore use a range of values for the function parameters.

[17] In both figures the planning horizon is truncated in the last few years. As is common in these types of dynamic models, towards the end of the planning period model results tend to exhibit extreme fluctuations.

conveyance infrastructure for ocean disposal is built and utilized, in all occasions and without exception, establishment and utilization of conveyance capacity towards the farming sector precedes the former, supporting our Propositions 1 and 2 above.

Figure 3 shows the quantities allocated to both sectors in the region. It can be noticed that for both crop alternatives (cotton and tomatoes), treated wastewater is being diverted to support agricultural production starting on day one. Another similar trend between the two crop scenarios is the gradual substitution of water types in agricultural irrigation, such that this sector becomes completely reliant on treated wastewater; however, with a difference in timing at which transition starts after nearly 50 and 85 periods for cotton and tomatoes, respectively. These allocation trends result in an increasing extraction path, which stabilizes rapidly at the level of recharge from precipitation and deep percolation of agricultural irrigation, so that the aquifer's steady-state storage level is at full capacity.

As mentioned, for cotton the transition from irrigating with groundwater to utilization of treated wastewater in agriculture is faster and happens earlier. The reason is the crop's relatively high salinity-tolerance, which allows for larger and earlier diversions of effluent, higher in salinity content (Figures 3 and 4), from the city's WWTP to the farming sector. This strategy prompts two processes generating regional benefits. The first is the increase in available income for the city, which results because effluent can be treated to a lower quality level without inflicting significant losses to the farming sector, in the case of cotton. The second process is the expansion of water consumption in the city, which becomes plausible due to reduction of agricultural water extractions from the CPR. Since tomatoes are less salinity-tolerant, allocations of higher saline effluent results in larger agricultural profits decreases, and therefore the diversion of treated wastewater to the farming sector under this scenario happens at a slower pace. This also results in a lower rate of groundwater quality degradation, such that the steady-state is reached 65 periods after the steady-state in the cotton scenario. The total effects of the differences in decisions between the two crop scenarios result in a regional welfare difference of 570 thousand USD annually in favor of the tomatoes scenario (not presented). A level of cautiousness is warranted when interpreting this result. One needs not assume that it suggests that tomato production as opposed to cotton would be the optimal crop choice for the region. Rather, it means that given both crops' production and costs characteristics, a regional management approach would yield greater regional net benefits when the farming sector specializes in a salt-sensitive crop as oppose to a salt-tolerant crop.

Finally, as previously suggested, we perform sensitivity analyses to our empirical model results. However, it is important to emphasize that this is not an attempt to prove results robustness to various changes in the entire set of parameters. As common in this type of models, sensitivity of results to parameter changes is a given. Furthermore, solution feasibility is not guaranteed throughout the possible range of parameters' values. We therefore focus our analysis on what we believe are two policy-relevant scenarios.[18] First, as it might appear that our theoretical outcome and proposition proofs hinge on the assumption that investment in remote disposal conveyance infrastructure is more expensive than investment in conveyance to the nearby farming sector, we investigate several scenarios assuming different possible investment proportions for the two disposal alternatives.

In our base scenario (A), both alternatives are not yet constructed at the initial conditions. We also assume for this scenario that investment in remote disposal is 20 times more expensive than investing in conveyance to the farming sector. Scenario B is the opposite of scenario A, in which conveyance to the nearby farming sector requires 20 times higher investment than the remote disposal site. In scenario C, investments are assumed to be equal to the average level of investment values in scenario A. In scenario D (E) the farming (remote disposal) alternative is already in place as the initial condition, and investment in the remote disposal (farming) alternative is set to the level assumed under scenario C. For the last scenario, F, we assume that both disposal alternatives' conveyance infrastructure are already in place. We report in Table 3 the differences in annual regional net welfare (in 10^3 USD) between the various scenarios, in which the reference for the differences calculated is scenario F.

As can be seen from Table 3, the differences in total welfare between scenarios are very small. It implies that the potential benefits from optimal central management of the region's water economy are similar, regardless of the initial setup of the region's infrastructure, or the proportions between investments of different effluent disposal alternatives. These scenarios could be distinguished into two groups, based on their predicted optimal trends. The first group includes scenarios B and E, in which investment in remote disposal is cheaper than in conveyance to the nearby farming sector. The second group is all other scenarios, in which the farming disposal alternative is either cheaper, or the two alternatives cost the same. Results differ between the groups, mainly in the optimal timing to start utilizing (either with or without

[18] From hereafter we assume that the farming sector grows only tomatoes.

investment associated) treated wastewater in the agricultural production. For the first group, this means diverting the effluent from the earlier preferred remote disposal, which translates to a timing decision done later than for the second group scenarios. Nevertheless, for all scenarios there exists such optimal timing, which strongly supports our theoretical findings and, even more so, rebuts the possibility that these findings rely heavily on the assumptions made regarding investment proportions between the two disposal alternatives.

Our second sensitivity analysis addresses the possibility of long-term changes of input intensity in crop production. It is already known that treated wastewater carries necessary nutrients for crop production, and therefore can potentially save on fertilizer use (Dawson and Hilton, 2011). Whether farmers would utilize more or less inputs, such as fertilizers and pesticides in the future, is unknown. Measuring the differences in net welfare between various scenarios of input intensiveness can therefore suggest how important policy intervention would be in either case. Our base scenario is notated now as scenario a. With respect to the current analysis, it is assumed that salinity contribution of other production inputs to the water quality in agriculture is fixed throughout the planning horizon at a low level. In scenario b, this contribution is also fixed over time, but at a high level. Scenarios c and d assume increasing and decreasing trends of input intensiveness, respectively (i.e., a gradual increase or decrease in the γ parameter, over the entire planning horizon). Table 4 reports the net welfare differences with respect to the base scenario a. Similar to our first analysis, differences are almost insignificant in magnitude. The policy conclusion is that the costs of optimal management to society would be almost equal, regardless of the input intensity strategy adopted by farmers in response to, or parallel to, utilization of treated wastewater in crop production.

Lastly, we find several additional worth-reporting results of the model's parameter changes. For example, we find that a more inelastic water demand in the city results in earlier diversions of effluent towards the farming sector, as oppose to the gradual transition reported in our base scenario. Assuming higher contaminant removal capabilities of the aquifer (i.e., changing the parameters of the decay rate function) yields intuitive outcomes, in which effluent diversion trends are more moderate and occur slower, with very little difference in all other aspects, compared with our base scenario.

4. Conclusions, Policy Implications, and Caveats

In this paper, we developed a framework that allows making optimal social decisions regarding productivity, welfare, and environmental health of water quantity-quality allocation in a regional context. We demonstrated the optimality of re-use of treated wastewater, using a social planner's approach, in which water quantity-quality allocations affect urban net income and agricultural productivity. Our theoretical as well as the empirical results suggest that of the various options facing the regional decision-maker, the use of treated wastewater for irrigation is the superior alternative for the region, as it maximizes the net regional benefits. However, we can expect private competitive solutions, that maximize individual or sectoral benefits, will depart from the social planner's solution. We should theoretically expect that such private solutions would result in inferior economic welfare of the entire region.

One clear conclusion from our analysis is that the strong interaction between the city treatment performance, the agricultural sector resilience, and the environment affect the optimal path and preferences among the investment alternatives. In terms of groundwater extraction, first order conditions suggest that in the optimal solution steady state extraction should equal the level of recharge (both from precipitation and agricultural water deep percolation). In terms of water quality, the optimal solution requires that the contaminant level would be set taking into account the pollution created by irrigated agriculture and the pollutant decay rate in the aquifer. These two findings support the need for policy interventions to address the dual quantity-quality regulation of water resources, especially, with a possible risk of water pollution. These outcomes are in agreement with previous work addressing the optimal-combined management of both groundwater quantity and quality dimensions (Hellegers et al., 2001; Roseta-Palma, 2002, 2003).

A relevant conclusion regarding the disposal location decision and the associated investment is that the city's problem, unlike the social planner's problem, does not account for possible externalities resulting from irrigating with lower- or higher-quality treated wastewater. Therefore, the city has no incentive to treat its wastewater above the regulated level. In such a case, it is possible that under alternative C (sending the WWTP effluent to irrigated agriculture) the city's private-optimal solution will include a positive level of taxes imposed on the city to account for the damages to the environment, resulting from disposal of remaining (not consumed by agriculture) effluent. That suggests our final conclusion that

Propositions 1 and 2 do not necessarily hold for the private problem of the city, but only for the social planner's problem.

Our regional model did not address several aspects. We did not include the stochastic nature of precipitation in the model and therefore our results might be downward biased vis-á-vis the value of the groundwater. Tsur and Graham-Tomasi (1991) showed analytically and estimated empirically the buffer value of groundwater under stochastic supply of surface water. They referred to the quantity dimension of groundwater as a source to balance scarcity effects of water availability. Considering the additional role of groundwater as a water quality-enhancing medium, would make our results even more significant. The aspects of the buffer value and the water quality-enhancing value of groundwater are left for our future research.

Another aspect that our model did not address is the extension of the social planner's solution, which is a feasibility test and maximization of regional benefits without considering the individual agent actions. That includes negotiations over the wastewater quality, price per unit between the city and the agricultural sector, and side payments among the agents (e.g., Dinar et al., 1986). Incorporating negotiated solutions into the theoretical and empirical frameworks we developed would add dimensions that are more practical in the context of multi-player groups that participate in the regional water reuse project.

Finally, our optimization model simplifies the farm-level operation. First, we consider only one agricultural decision-maker in the region. Second, we consider a farming operation with only one crop instead of multi-crop farm, which could add more flexibility to the on-farm decisions. These three aspects, which have not been part of the theoretical and empirical social planner's model, will be included in a future regional model that will be developed for the Escondido region in California.

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Table 1- Parameters for utility function in the city and agricultural production

Parameter	Value		Description/Units
City			
D_u [a]	125		
η_1 [b]	-0.51		
η_2 [b]	0.43		
Agriculture			
	Cotton	Tomato	
$\overline{ev}$	2.39	1.97	Maximum Evapotranspiration
α_1	0.000013	0.0011	
α_2	47.06	21.85	
α_3	-0.99	-1.47	
α_4	3.14	2.37	
b_1	0.6	37.38	
b_2	-0.12	0	
$\underline{ev}$	0.47	0.66	Minimal required evapotranspiration for crop production
P^Y	1586.2	43.2	Crop price ($/ton)
$\tilde{w}$ [c]	2.9	2.7	Observed average water application (acre-feet/acre)
f_0	631.58	76.36	$/acre
f_1	0.29	1.33	$/acre^2

[a] This is the value for the base period; [b] Reported mean value in Espey et al. (1997) and Dalhuisen et al. (2003); [c] Source: USDA, 2013 Farm and Ranch Irrigation Survey(FRIS), Table 36, http://www.agcensus.usda.gov/Publications/2012/Online_Resources/Farm_and_Ranch_Irrigation_Survey/

Table 2- Groundwater parameters

Parameter	Value	Units	Description
AR	4942	Acres	Aquifer Area
S	0.1		Storage Coefficient/Specific Yield
ec	0.002	$/$10^3$ acre-feet	Per unit of volume pumping cost
θ	0.1		Irrigation return rate
δ_0	0.1		Decay rate function constant
δ_1	0.05		Decay rate slope parameter with respect to groundwater storage level change
ρ	0.8		Power at which storage level is raised by in the decay rate function
γ	0.51	dS/m	Salinity level induced by input use in agriculture
e	0.7		Agriculture contamination function parameter

Table 3- Annual net welfare differences between conveyance-investment scenarios

Scenario	Welfare difference with respect to scenario F (10^3 USD)
A	-5.62
B	-2.88
C	-18.20
D	-5.25
E	-1.44

Note: A- base scenario, higher investment in remote disposal; B- higher investment in faming disposal; C- equal investments in both alternatives; D- existing infrastructure to agriculture; E- existing infrastructure to remote disposal; F- existing infrastructure to both disposal alternatives.

Table 4- Annual net welfare differences between input-intensive scenarios

Scenario	Welfare difference with respect to scenario a (10^3 USD)
b	-6.06
b	-1.12
d	-1.35

Note: a- base scenario, fixed low level of input intensity; b- fixed high level of input intensity; c- increasing trend in input intensity; d- decreasing trend in input intensity.

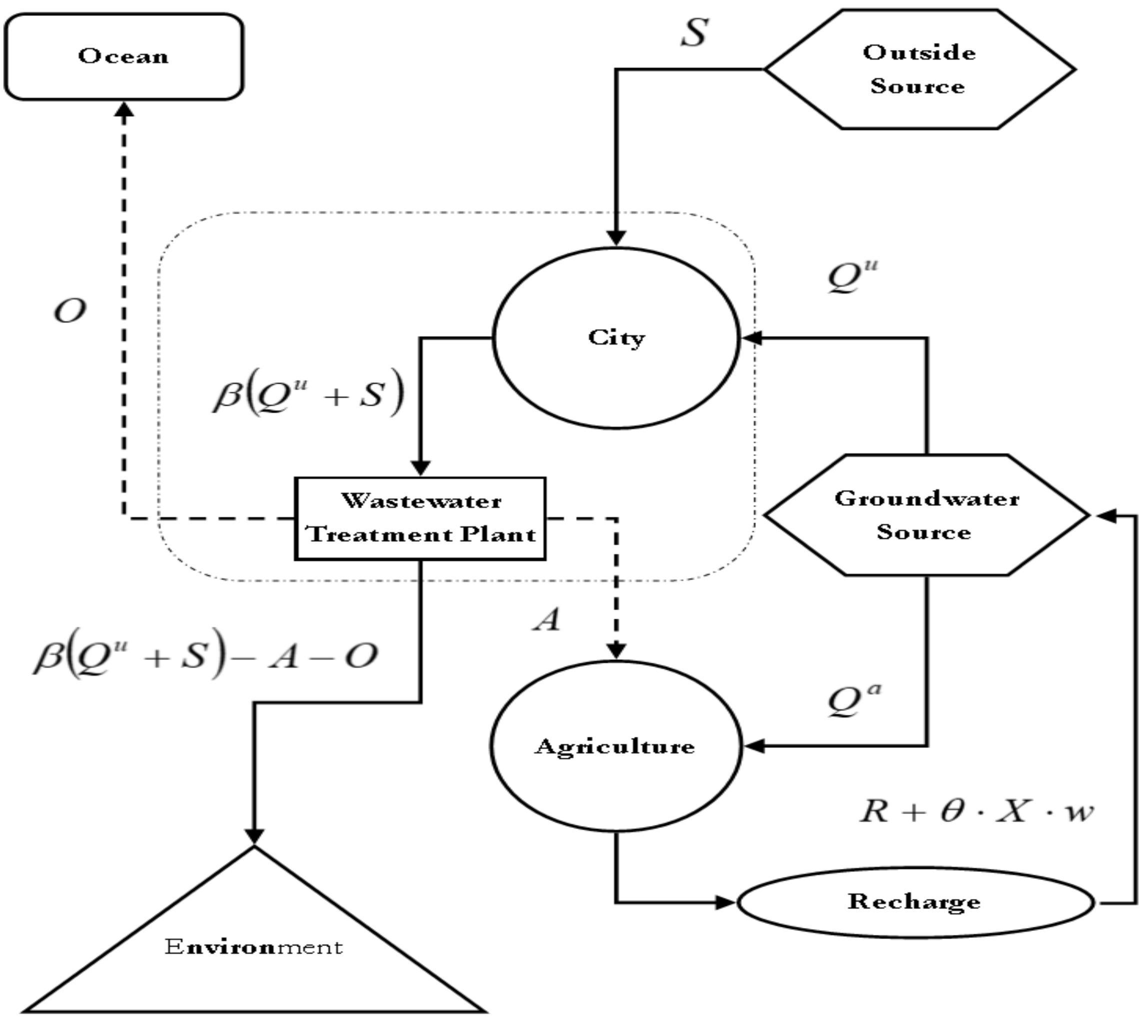

Figure 1: Schematic regional setting

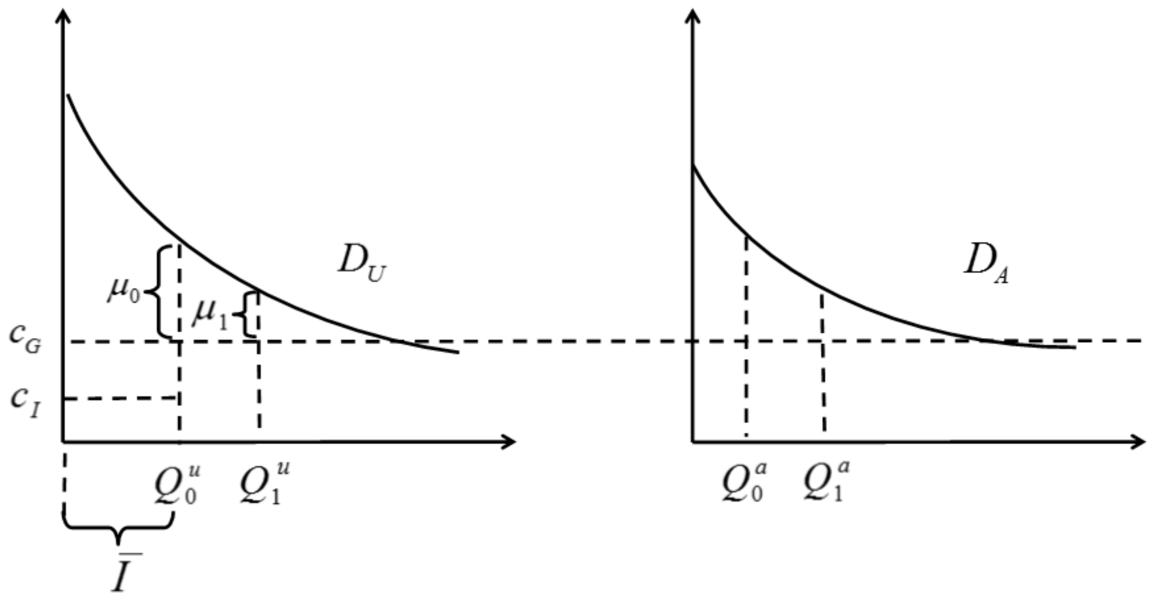

Figure 2: Static optimal allocation illustration

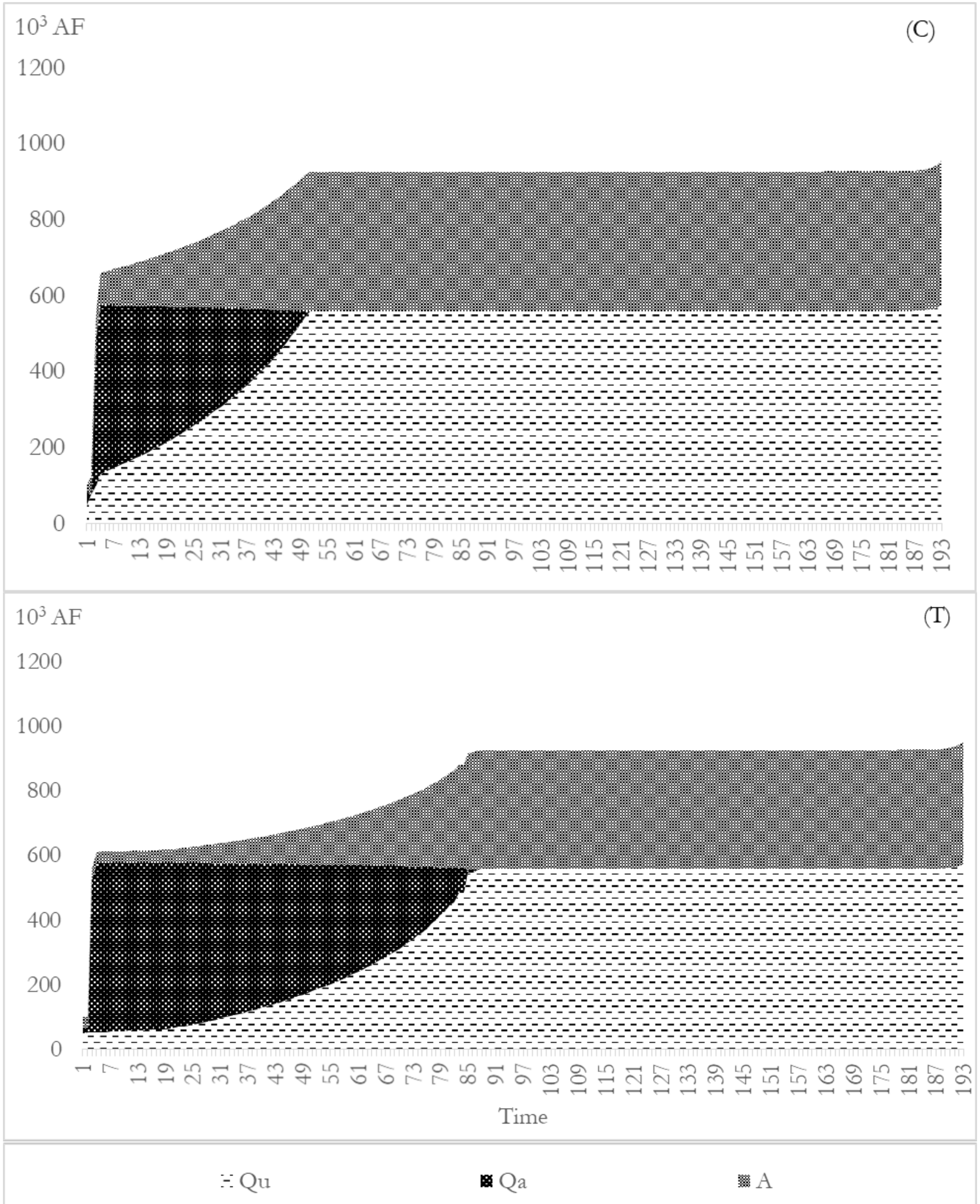

Figure 3: Water allocation between sectors for a cotton (C) and tomatoes (T)-based farming sector (10^3 acre-feet/year).

Note: Q_u - Quantity consumed in the city and extracted from the groundwater CPR; Q_a - Quantity consumed in agriculture and extracted from the groundwater CPR; A - Quantity of effluent consumed in agriculture and conveyed from the WWTP;

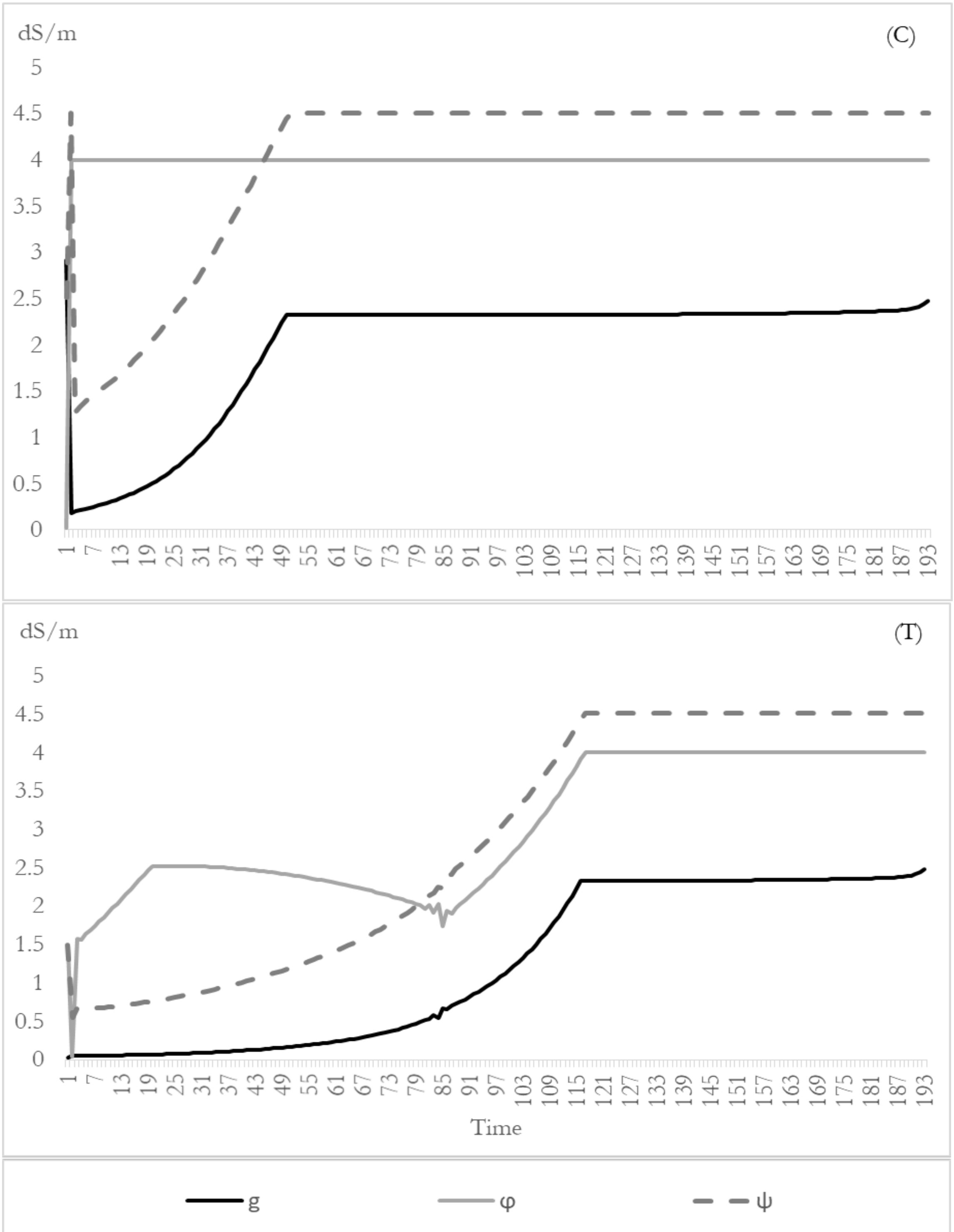

Figure 4: Groundwater, irrigation, and WWTP effluent quality level for a cotton (C) and tomatoes (T)-based farming sector (dS/m).

Note: g - Groundwater quality level; φ - Quality of treated wastewater; ψ - Quality level of applied water in agriculture.

Appendix A

As explained earlier in the text, we describe the social planner welfare maximization problem for each alternative (A, B, and C) separately, assuming each case prevails for an infinite horizon. We denote the problems A1, A2, and A3, respectively, deriving for each its necessary first-order conditions (FOC), and the resulting optimal solution.

Let the regional social planner welfare maximization problem (A1) be:

$$\text{(A1)} \quad \underset{Q^u,S,Q^a,X,w,\varphi}{Max} \int_0^\infty e^{-rt} \cdot \Big[U\big\{ I\big(\beta \cdot (Q^u(t)+S(t)), \varphi(t)\big), Q^u(t)+S(t) \big\} + P^Y(t)\cdot X(t)\cdot Y\{w(t),\psi(g(t)); r(t)\} - f(X(t)) - C(G(t))\cdot\big(Q^u(t)+Q^a(t)\big) - h(g(t))\cdot Q^u(t) - D\big(E(t),\varphi(t)\big)\Big]dt$$

$s.t.$

a) $\dot{G} = R(t) + \theta \cdot X(t)\cdot w(t) - Q^u(t) - Q^a(t)$

b) $\dot{g} = e(\psi(g(t))) - \delta(G(t))\cdot g(t)$

c) $\varphi(\mathrm{t}) \le \overline{\varphi}$

d) $X(t)\cdot w(t) \le Q^a(t)$

e) $S(t) \le \overline{S}$

f) $G(0) = G_0$

g) $g(0) = g_0$

Let the respective Lagrangian function be:

$$L = U\big\{ I\big(\beta \cdot (Q^u(t)+S(t)), \varphi(t)\big), Q^u(t)+S(t)\big\} + P^Y(t)\cdot X(t)\cdot Y\{w(t),\psi(g(t)); r(t)\} - f(X(t)) - C(G(t))\cdot\big(Q^u(t)+Q^a(t)\big) - h(g(t))\cdot Q^u(t) - D\big(E(t),\varphi(t)\big) + m_1(t)\cdot\big(R(t)+\theta\cdot X(t)\cdot w_t - Q^u(t) - Q^a(t)\big) + m_2(t)\cdot\big(e(\psi(g(t))) - \delta(G(t))\cdot g(t)\big) + \lambda_\varphi\cdot(\overline{\varphi}-\varphi(t)) + \lambda_W\cdot\big(Q^a(t) - X(t)\cdot w(t)\big) + \lambda_S\cdot\big(\overline{S} - S(t)\big)$$

The FOC, $\frac{\partial L}{\partial z_{A1}} = 0$ where $z_{A1} \in \{Q^u(t), S(t), Q^a(t), \varphi(t), X(t), w(t)\}$, and

$\dot{m}_i + r \cdot m_i(t) = -\frac{\partial L}{\partial s}$ where $i = 1,2$; $s \in \{G(t), g(t)\}$, along with the Karush-Kuhn-Tucker

conditions $\frac{\partial L}{\partial \lambda_j} \lambda_j = 0$, $\lambda_j \geq 0$; $j = \varphi, W, S$ are re-organized and presented as follows:

(A1.1) $\lambda_W(t) = C(G(t)) + m_1(t)$

(A1.2) $\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot D_E + m_1(t)$

(A1.3) $\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot D_E + \lambda_S(t)$

(A1.4) $\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi$

(A1.5) $P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + (\lambda_W(t) - \theta \cdot m_1(t)) \cdot w(t)$

(A1.6) $X(t) \cdot (P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)) = 0$

(A1.7) $\dot{m}_1 - r \cdot m_1(t) = \underbrace{C'(G(t))}_{\substack{\text{Marginal Cost}\\ \text{of Extraction (-)}}} \cdot (Q^u + Q^a) + m_2(t) \cdot \underbrace{\delta'((G(t))}_{\substack{\text{Marginal}\\ \text{Decay Rate (−)}}} \cdot g(t)$

(A1.8) $\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\substack{\text{Marginal}\\ \text{Contamination Rate (+)}}} - \delta(G(t)) - r \right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\substack{\text{Marginal Costs of}\\ \text{Groundwater Treatment}\\ \text{for Urban Consumption (+)}}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\substack{\text{VMP of Applied}\\ \text{Water Quality}\\ \text{in Agriculture (-)}}}$

(A1.9) $\lambda_\varphi(t) \cdot (\overline{\varphi} - \varphi(t)) = 0$, $\lambda_\varphi(t) \geq 0$

(A1.10) $\lambda_W(t) \cdot (Q^a(t) - X(t) \cdot w(t)) = 0$, $\lambda_W(t) \geq 0$

(A1.11) $\lambda_S(t) \cdot (\overline{S} - S(t)) = 0$, $\lambda_S(t) \geq 0$

Equation (A1.1) states that the shadow value associated with the available water constraint for irrigation should be equal to the sum of the unit cost of extraction and the scarcity rent, and therefore will always be positive in the optimal solution. Equations (A1.2) and (A1.3) equate the marginal utility of water consumption to the marginal cost associated with the use of each water source—groundwater and the outside source, respectively. For the case of groundwater, according to (A1.2), this marginal cost will be the sum of extraction cost, treatment cost (in case that the groundwater quality falls under the threshold permitted for drinking)[19], the marginal damage associated with discharging an additional unit of effluent to the environment, and the scarcity rent. In the other case, the marginal utility of consuming one more unit from the outside source should only equate to the marginal damage and the shadow value associated with that source's availability constraint. It immediately follows from these two equations that when the outside source is available in a very large amount (i.e., $\lambda_S = 0$), the optimal consumption in the city will always be based on the outside source alone. In (A1.4) marginal utility from treating effluent to a lower quality is equated with the sum of the marginal damage associated with discharging water at a lower quality to the environment and the shadow value of the regulatory quality standard constraint. Substituting equation (A1.1) into (A1.5) and (A1.6), and given that there exists an internal solution with respect to cultivable land (i.e., $X^*(t) > 0 \ \forall t$), yields the following respectively:

(A1.12) $P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + \left[\underbrace{C(G(t)) + (1-\theta) \cdot m_1(t)}_{\text{VMP of Water Applied}(+)} \right] \cdot w(t)$

(A1.13) $P^Y(t) \cdot Y_w = C(G(t)) + (1-\theta) \cdot m_1(t)$

The latter equates the value of marginal product in agriculture of one more unit of water applied for irrigation to its associated marginal cost, which is the sum of the unit cost of extraction and the scarcity rent, multiplied by one, minus the percolation rate—accounting for that unit contribution to the groundwater stock level. Equation (A1.12) refers to the other input used in agricultural production (in our model), which is land, and requires the identity of the marginal value of unit of land to be the marginal cost associated with the cultivation of it. That cost is equal to the marginal crop production cost associated with different input use (e.g., fertilizer and labor), and expressed by the function $f(\cdot)$, plus the value of water used to irrigate one more unit of land. Equation (A1.7) defines the optimal path for scarcity rent evolvement over time. Naturally, it is dependent upon the marginal cost of extraction,

[19] It can be observed that in the social planner optimal solution, $h(g^*(t))$ will always be set to zero, that is if the initial groundwater quality g_0 is of better quality than the threshold $\bar{g}$. This is easy to show, as lower water quality doesn't have any positive effect on the objective function. It doesn't mean however that $g^*(t) \geq \bar{g}$ is not a feasible solution.

multiplied by the extraction quantities, but it is also related to the opportunity costs resulting from groundwater quality degradation. The evolvement of the latter is defined in equation (A1.8), and is associated with the effect of groundwater quality on both sectors, i.e., the marginal cost of groundwater treatment to the city, and the value of marginal productivity of water quality in agriculture. The net discounted effect on groundwater quality, which is expressed as the sum of marginal rate of contamination from water irrigation, the decay rate, and the interest rate will also affect the optimal trajectory of this co-state. Equations (A1.9) through (A1.11) are the usual Karush-Kuhn-Tucker conditions, and dictate that the shadow value of any binding constraint in the optimization must be non-negative.

We move on to describe the planner's problem when effluent could be discharged from the WWTP to the ocean at a pre-determined cost. This problem is denoted (A2) as follows:

(A2)
$$\underset{Q^u,S,Q^a,O,X,w,\varphi}{Max}\int_0^\infty e^{-rt}\cdot\Big[U\big\{I\big(\beta\cdot(Q^u(t)+S(t)),\varphi(t)\big),Q^u(t)+S(t)\big\}$$
$$+P^Y(t)\cdot X(t)\cdot Y\{w(t),\psi(g(t));r(t)\}-f(X(t))$$
$$-C(G(t))\cdot\big(Q^u(t)+Q^a(t)\big)-h(g(t))\cdot Q^u(t)-\nu(O(t))-D(E(t),\varphi(t))\Big]dt$$

$s.t.$

a) $\dot{G}=R(t)+\theta\cdot X(t)\cdot w(t)-Q^u(t)-Q^a(t)$

b) $\dot{g}=e(\psi(g(t)))-\delta(G(t))\cdot g(t)$

c) $\varphi(\mathrm{t})\le\overline{\varphi}$

d) $X(t)\cdot w(t)\le Q^a(t)$

e) $S(t)\le\overline{S}$

f) $O(t)\le\beta\cdot(Q^u(t)+S(t))$

g) $G(0)=G_0$

h) $g(0)=g_0$

Problem (A2) is slightly different than problem (A1) in that it also includes the cost of conveyance to the ocean as a function of quantity, as part of the objective function. Constraint (f) is introduced to ensure that conveyance to the ocean will be limited to the amount of available effluent. Tradeoffs between the optimal solutions of these two problems can be explained intuitively. Net benefits will only accrue to the region in the following instances: a) if the costs of conveying the effluent to the ocean are lower than the avoidable damage; b) if the city earns from treating its effluent to a lower quality, and c) if consuming more water in the city and discharging it to the ocean (avoiding the associated environmental damage) exceeds the losses to farmers from diverting shared resource water to the city. The corresponding FOC for problem (A2) are presented below:

(A2.1) $\lambda_W(t) = C(G(t)) + m_1(t)$

(A2.2) $$\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot (D_E - \lambda_E(t)) + m_1(t)$$

(A2.3) $$\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot (D_E - \lambda_E(t)) + \lambda_S(t)$$

(A2.4) $$\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi$$

(A2.5) $P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + (\lambda_W(t) - \theta \cdot m_1(t)) \cdot w(t)$

(A2.6) $X(t) \cdot (P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)) = 0$

(A2.7) $$\dot{m}_1 - r \cdot m_1(t) = \underbrace{C'(G(t))}_{\substack{\text{Marginal Cost} \\ \text{of Extraction (-)}}} \cdot (Q^u + Q^a) - m_2(t) \cdot \underbrace{\delta'((G(t))}_{\substack{\text{Marginal} \\ \text{Decay Rate (-)}}} \cdot g(t)$$

(A2.8) $$\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\substack{\text{Marginal} \\ \text{Contamination Rate (+)}}} - \delta(G(t)) - r \right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\substack{\text{Marginal Costs of} \\ \text{Groundwater Treatment} \\ \text{for Urban Consumption (+)}}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\substack{\text{VMP of Applied} \\ \text{Water Quality} \\ \text{in Agriculture (-)}}}$$

(A2.9) $\lambda_\varphi(t) \cdot (\overline{\varphi} - \varphi(t)) = 0$, $\lambda_\varphi(t) \geq 0$

(A2.10) $\lambda_W(t) \cdot (Q^a(t) - X(t) \cdot w(t)) = 0$, $\lambda_W(t) \geq 0$

(A2.11) $\lambda_S(t) \cdot (\overline{S} - S(t)) = 0$, $\lambda_S(t) \geq 0$

(A2.12) $\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - O(t)) = 0$, $\lambda_E(t) \geq 0$

(A2.13) $\lambda_E(t) = D_E - v'(O(t))$

These first-order conditions are quite similar to the ones from problem (A1), and therefore do not require the same detailed description as presented above. It is, however, important to notice the difference between equations (A2.2) and (A2.3), and their counterparts (A1.2) and (A1.3) in the previous problem. These two equations equate marginal utility from water consumption in the city to their respective marginal cost, according to the source of supply. It is easy to observe that in the private case where λ_E, which is the shadow value associated with the effluent availability constraint, equals to zero, these two equations would become identical to (A1.2) and (A1.3). Equation (A2.13) depicts the value for that shadow value, and defines it as equal to the difference between the marginal damage and marginal conveyance cost to the ocean. The logic is simple: diverting water from the environment to the ocean is only worthwhile as long as the costs to society are lower than the damage avoided. Equation (A2.12) states that when that difference is positive, the constraint must be binding, which means that all effluent should be discharged at the ocean. An interesting phenomenon arises in equation (A2.13) with respect to the relationship between optimal solution of the system, and the predetermined assumptions regarding the damage function. It can be seen that if one follows the assumption of Horan (2001) (i.e., the damage function is non-decreasing and convex), the optimal solution will never suggest discharging all the effluent to the ocean. The reason is that when this occurs, the marginal damage avoided will be very small, and marginal cost of conveyance will then be higher (as $\nu(\cdot)$ is also assumed to be a non-decreasing and convex function). That translates to a negative λ_E which cannot be an optimal solution according to (A2.12). When the damage function is non-decreasing and concave (or linear), the decision to divert all effluent to the ocean is always the optimal solution, and the shadow value of effluent availability constraint must be positive.

Problem (A3) addresses the case in which the planner faces the alternative to either discharge the treated wastewater to the environment or divert it for irrigation of crops in a neighboring agricultural district. It is presented as follows:

$$\text{(A3)} \quad \underset{Q^u,S,Q^a,A,X,w,\varphi}{Max} \int_0^\infty e^{-rt} \cdot \Big[U\big\{ I\big(\beta \cdot (Q^u(t)+S(t)), \varphi(t)\big), Q^u(t)+S(t) \big\} + P^Y(t)\cdot X(t)\cdot Y\big\{ w(t), \psi\big(g(t), \varphi(t), A(t), Q^a(t)\big), r(t) - f\big(X(t)\big) \big\} - C\big(G(t)\big)\cdot\big(Q^u(t)+Q^a(t)\big) - h\big(g(t)\big)\cdot Q^u(t) - \upsilon\big(A(t)\big) - D\big(E(t), \varphi(t)\big) \Big] dt$$

$s.t.$

a) $\dot{G} = R(t) + \theta \cdot X(t) \cdot w(t) - Q^u(t) - Q^a(t)$

b) $\dot{g} = e(\psi(g(t), \varphi(t))) - \delta(G(t)) \cdot g(t)$

c) $\varphi(\mathrm{t}) \le \overline{\varphi}$

d) $X(t) \cdot w(t) \le Q^a(t) + A(t)$

e) $S(t) \le \overline{S}$

f) $A(t) \le \beta \cdot (Q^u(t) + S(t))$

g) $G(0) = G_0$

h) $g(0) = g_0$

Problem (A3) resembles problem (A2) in the sense that it also includes an alternative to environmental damage from effluent discharge. However, there is a distinct difference between the two problems, as the current one facilitates effluent quality effects on agricultural productivity and groundwater quality evolvement over time. We also expand the definition of the applied water quality function in order to account for the effects of water blending from the different sources, as mentioned in the text (see footnote 10). This expansion dictates that:

(A4) $\psi_A, \psi_{Q^a} > 0; \psi_{A\varphi} = \psi_{\varphi A} > 0; \psi_{Q^a g} = \psi_{gQ^a} > 0; \psi_{AA}, \psi_{Q^a Q^a} \psi_{gA}, \psi_{\varphi Q^a} < 0$

(A5) $$\begin{cases} \psi_{AQ^a} \le 0 & \text{if} \quad \varphi \ge \dfrac{A^2}{Q^a \cdot (2A + Q^a)} + 1 \\ \psi_{AQ^a} > 0 & \text{else} \end{cases}$$

(A6) $$\begin{cases} \psi_{Q^a A} \le 0 & \text{if} \quad g \ge \dfrac{{Q^a}^2}{A \cdot (2Q^a + A)} + 1 \\ \psi_{Q^a A} > 0 & \text{else} \end{cases}$$

To summarize, the weighted average functional form simply dictates that the average quality is positively affected by the usage of each source, and that the effect diminishes as more of that source is used. Equations (A5) and (A6) imply that the cross derivative of the average quality with respect to the quantities consumed in agriculture is changing its sign, according to the proportions between effluent and groundwater use. As in problem (A2), the costs of effluent conveyance to the agricultural district are considered explicitly in the objective function. Equation (f) is similar to its counterpart in problem (A2) and limits the conveyed quantity to irrigation by the available effluent volume. The FOC for problem (A3) are presented below:

(A3.1) $\lambda_W(t) = C(G(t)) + m_1(t) - \underbrace{Y_\psi \psi_{Q^a}}_{(-)}$

(A3.2) $\underbrace{\underbrace{U_I I_{Q^u}}_{\text{Indirect Effect (-)}} + \underbrace{U_{Q^u}}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = C(G(t)) + h(g(t)) + \beta \cdot (D_E - \lambda_E(t)) + m_1(t)$

(A3.3) $\underbrace{\underbrace{U_I I_S}_{\text{Indirect Effect (-)}} + \underbrace{U_S}_{\text{Direct Effect (+)}}}_{\text{Marginal Utility (+)}} = \beta \cdot (D_E - \lambda_E(t)) + \lambda_S(t)$

(A3.4) $\underbrace{U_I I_\varphi}_{\text{Marginal Utility (+)}} = \underbrace{D_\varphi}_{\text{Marginal Damage (+)}} + \lambda_\varphi - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_\varphi}_{\text{VMP of Applied Water Quality in Agriculture (-)}} - m_2(t) \cdot \underbrace{e_\psi \psi_\varphi}_{\text{Marginal Contamination Rate (+)}}$

(A3.5) $P^Y(t) \cdot Y\{w(t), g(t); r(t)\} = f'(X(t)) + (\lambda_W(t) - \theta \cdot m_1(t)) \cdot w(t)$

(A3.6) $X(t) \cdot (P^Y(t) \cdot Y_w + \theta \cdot m_1(t) - \lambda_W(t)) = 0$

(A3.7) $\dot{m}_1 - r \cdot m_1(t) = \underbrace{C'(G(t))}_{\text{Marginal Cost of Extraction (-)}} \cdot (Q^u + Q^a) + m_2(t) \cdot \underbrace{\delta'((G(t))}_{\text{Marginal Decay Rate (+)}} \cdot g(t)$

(A3.8) $\dot{m}_2 + \left[\underbrace{e_\psi \psi_g}_{\text{Marginal Contamination Rate (+)}} - \delta(G(t)) - r \right] \cdot m_2(t) = \underbrace{h'(g(t))}_{\text{Marginal Costs of Groundwater Treatment for Urban Consumption (+)}} \cdot Q^u(t) - X(t) \cdot \underbrace{P^Y(t) \cdot Y_\psi \psi_g}_{\text{VMP of Applied Water Quality in Agriculture (-)}}$

(A3.9) $\lambda_\varphi(t) \cdot (\overline{\varphi} - \varphi(t)) = 0, \lambda_\varphi(t) \geq 0$

(A3.10) $\lambda_W(t) \cdot (Q^a(t) + A(t) - X(t) \cdot w(t)) = 0, \lambda_W(t) \geq 0$

(A3.11) $\lambda_S(t) \cdot (\overline{S} - S(t)) = 0, \lambda_S(t) \geq 0$

(A3.12) $\lambda_E(t) \cdot (\beta \cdot (Q^u(t) + S(t)) - A(t)) = 0, \lambda_E(t) \geq 0$

(A3.13) $\lambda_E(t) = \lambda_W(t) + D_E + \underbrace{Y_\psi \psi_A}_{(-)} - \upsilon'(A(t))$

The differences between problem (A2) and the current one are well summarized in equation (A3.4). It is the last two components on the right-hand side of the equation that tell the story. The third component is the marginal effect of a change in effluent quality on agricultural productivity. The fourth component is the opportunity cost of groundwater quality degradation, multiplied by the marginal rate of contamination resulting from percolation of irrigation water. One can notice that an optimal solution of higher quality level of effluent (with respect to both problems (A1) and (A2)) can be reached, as both effects contribute to the same direction (let us denote this optional solution as z_{A3}^{u} to be used in the proof that follows). Considering the expansion introduced above regarding the quality of water applied in agriculture, we note that the only effect is through the change in shadow values of the water availability constraint to the agriculture sector (A3.1), and the one associated with the effluent availability constraint (A3.13). Recall that $Y_{\psi\psi} > 0$ and that $\psi_{Q^a Q^a} < 0$, therefore equation (A3.1) means that the net benefit associated with releasing constraint (d) by one unit is higher, either when quality is better, or when supply of groundwater to the farming sector is lower. For equation (A3.13), the interpretation is rather similar. Effluent high in contaminant levels affect $\lambda_E(t)$ negatively, and therefore are less attractive for the farming sector in the optimal solution. Another point worth mentioning is related to equation (A3.13). Unlike its counterpart in problem (A2), the shadow value of effluent availability constraint is equated not only to the difference between marginal damage and marginal conveyance costs, but also $\lambda_W(t)$ (the shadow value of water availability for irrigation constraint) is added on the right-hand side. As already discussed previously, this shadow value is positive. Therefore, in the optimal solution for problem (A3), as opposed to (A2), even when the damage function is considered to be non-decreasing and convex, there can be a situation in which all effluent is diverted away from the environment.

Proposition 1. *Facing identical functional forms and sets of parameters*

$$V_{A3}\left(z_{A3}^{*}\right) \geq V_{A2}\left(z_{A2}^{*}\right) \geq V_{A1}\left(z_{A1}^{*}\right)$$

Proof. We first derive the relationship between problems (A1) and (A2). As described above, the optimal conditions for problem (A1) can be represented as a private case of the optimal conditions solving problem (A2). It immediately follows that $Z_{A1} \in Z_{A2}$, and therefore $z_{A1}^{*} \in Z_{A2}$, but also that $V_{A2}\left(z_{A1}^{*}\right) = V_{A1}\left(z_{A1}^{*}\right)$. It then follows that for every given empirical setting, if $z_{A2}^{*} \neq z_{A1}^{*}$ then it must be that $V_{A2}\left(z_{A2}^{*}\right) \geq V_{A1}\left(z_{A1}^{*}\right)$. The relationship between problems (A2) and (A3) is less trivial and involves using a proof of contradiction. Therefore, let as assume that $V_{A2}(z^*_{A2}) \geq V_{A3}(z^*_{A3})$ for every $z^*_{A2} \in Z_{A2}$ and $z^*_{A3} \in Z_{A3}$, from the first part of the proof it follows that $Z_{A3} \in Z_{A2}$. However, as mentioned above $z^u_{A3} \notin Z_{A2}$, and obviously $z^u_{A3} \in Z_{A3}$, which means a contradiction.

We continue by notating $z^*_{i\tau}$ as the optimal solution for any sub-period $\tau = [\underline{T}, \overline{T}] \in \{0, \infty\}$, where again $i \in \{A1, A2, A3\}$. We also notate $V^\tau_i(z^*_{i\tau})$ as the maximum value resulting from choosing $z^*_{i\tau}$ over τ.

Proposition 2. *Let alternative A set the initial conditions for a regional social welfare planner facing the two alternatives for effluent discharge B and C. Then,*

$$if \;\; F_B > F_C \Rightarrow T^*_C < T^*_B \Rightarrow T^*_B \in \{\varnothing\}$$

Proof. We had already shown that $V_{A3}(z^*_{A3}) \geq V_{A2}(z^*_{A2}) \geq V_{A1}(z^*_{A1})$, it follows therefore, that for every sub-period τ it must also hold that $V^\tau_{A3}(z^*_{A3\tau}) \geq V^\tau_{A2}(z^*_{A2\tau}) \geq V^\tau_{A1}(z^*_{A1\tau})$. It also holds that for an alternative $j \in \{B, C\}$ to have an optimal time $T^*_j \in \{0, \infty\}$ to invest in, it must satisfy the condition that $V^{\tau_T}_{i(j)}(z^*_{i(j)\tau_T}) \geq V^{\tau_T}_0(z^*_{0\tau_T}) + F_j$, where $i(j)$ stands for the problem $i \in \{A2, A3\}$ corresponding to alternative j, and $\tau_T = [T^*_j, \infty)$. The notation '0' stands for the problem associated with the existing alternative at the time of investment. Now, assume that there exists an optimal time to invest in each of the alternatives B and C such that $T^*_B, T^*_C \in \{0, \infty\}$. Remember that $V^\tau_{A3}(z^*_{A3\tau}) \geq V^\tau_{A2}(z^*_{A2\tau})$, and by assumption that $F_B > F_C$, it means that for any given τ_T it holds that $V^{\tau_T}_{A3}(z^*_{A3\tau_T}) - F_C > V^{\tau_T}_{A2}(z^*_{A2\tau_T}) - F_B$, which in turn implies that $T^*_C < T^*_B$. But if this is the case, then for alternative B to have an optimal timing for investment it must hold that $V^{\tau_T}_{A2}(z^*_{A2\tau_T}) \geq V^{\tau_T}_{A3}(z^*_{A3\tau_T}) + F_B$, which contradicts $V^\tau_{A3}(z^*_{A3\tau}) \geq V^\tau_{A2}(z^*_{A2\tau})$, and therefore we can conclude that when investment in alternative C had already occurred $T^*_B \in \{\varnothing\}$.